A decentralized observer for a general class of Lipschitz systems

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Abstract—The paper presents a decentralized observer for a class of multi-agent systems. The proposed observer allows each robot of the team to estimate the overall system state provided that the communication network is connected and that the motion control law is Lipschitz. In case of perfect measurements the observer error is proved to be exponentially convergent to zero, while global uniform ultimately boundness is proved for the case of bounded non-vanishing noise on the state measurements. The approach is validated via numerical simulations considering, as a case study, the decentralized control of the centroid and the formation of a team of robots.

I. INTRODUCTION

The research on mobile multi-agent systems received increasing attention in the last decades due to their potentiality in term of redundancy, flexibility, and scalability, and to their wide application domain. For example, mobile multi-agent systems present several key features that make them well suitable for logistics applications, as transportation of materials or warehousing. The on-board intelligence of the mobile agents, indeed, can be used in dynamic scenarios to dynamically plan and re-plan the motion strategy of the overall system to properly accomplish the assigned mission. Such characteristics make the multi-agent system more flexible with respect to Autonomous Guided Vehicles (AGVs) with pre-assigned paths, and allow the implementation of dynamic optimization strategies to reduce timing and costs of mission executions. To increase the flexibility while reducing system’s costs, the lack of a centralized coordination unit is a desired feature; thus each agent is allowed only to communicate with its direct neighbours (i.e., the agents within its communication range) and it is required to take its decisions only on the base of its local information. Since, on the other side, the optimization of the individual motion strategies may depend on the overall system state, it might be useful for each agent to have an estimate of this state, even if not it is directly connected to all the agents in the team. Thus, in this paper we present a decentralized state observer for multi-agent system that can be used for a general class of Lipschitz systems; the proposed observer allows each agent to estimate the overall system state provided that the communication network is connected and that the motion control law is Lipschitz.

The problem addressed in this work can be framed in the more general area of motion control of distributed networked agent systems. The literature related to this field deals with both cooperative control and communication problems such as controlling the centroid of the multi-agent system, its variance, and its orientation [5], the cooperative team reconfiguration in response to measured information of the environment [13], control of unmanned air vehicles [21], flocking [23], formation control [24], [9], rendezvous and formation control with connectivity maintenance [14], the design of sensor networks [7], the multi-robot patrolling [17], [18]. An overview on control issues for distributed and networked team of agents can be found in [6], [11], [16].

Several works from the literature deal with the problem of developing local control and communication strategies to manage global properties of the system; a representative example is the consensus problem, i.e. the problem of making the agents reaching an agreement regarding a certain variable dependent on the state of all the others, where each agent can only communicate with its direct neighbors [19], [20]. An attempt to control a collective variable expressed in terms of formation statistics (e.g. the centroid and the shape of the team) by resorting to a distributed estimator of the actual collective variable can be found in [10] and [25]. Decentralized estimation and control are also investigated in [22] in the framework of linear state feedback control, and in [8] in the context of sensory networks, where the estimated state covers the role of solving surveillance and monitoring tasks.

The paper builds on the results of [1], [4], where tracking of the sole weighted centroid has been achieved by resorting to a distributed controller, and of [2], [3] where the tracking of an assigned time-varying relative formation, in addition to the centroid, is achieved. The paper extends such previous works by generalizing the observer to work with a wider class of systems, i.e. for mobile robots characterized by more general dynamic model and control law, and where the state measurements are corrupted by noise. The proposed approach is validated via numerical simulations in the case of a second order multi-agent system performing centroid and formation tracking control.

The paper is organized as follows. In Section II, the main symbols and variables are defined. In Section III, the decentralized observer is presented whose stability is proved in Section IV. To show the utility of the approach in a practical case, the estimated collective state is used to implement the decentralized centroid and formation control of a team of
mobile agents as described in Section V. The effectiveness of the approach is shown by means of simulations in Section VI.

II. BACKGROUND

Consider a system composed of $N$ agents. The state of the $i$th agent is denoted by $x_i \in \mathbb{R}^n$. The agents are assumed to have the following non-linear dynamics

$$\dot{x}_i = f(t, x_i, u_i),$$

(1)

where $u_i \in \mathbb{R}^m$ is the control input. In the following is required that function $f$ satisfies the following Lipschitz condition

$$\|f(t, x', u') - f(t, x, u)\| \leq L_{f,x} \|x' - x\| + L_{f,u} \|u' - u\|$$

(2)

for all $x, x', u, u' \in \mathbb{R}^n$, where $L_{f,x}, L_{f,u} > 0$ are the Lipschitz constants and where $\|\cdot\|$ represents the 2-norm of its argument.

The vector $x = [x_1^T \ldots x_N^T]^T \in \mathbb{R}^{Nn}$ represents the collective state whose dynamics is

$$\dot{x} = F(x, u),$$

(3)

where $F = [f(t, x_1, u_1)^T \ldots f(t, x_N, u_N)^T]^T \in \mathbb{R}^{Nn}$.

It is supposed that each agent is able to access a noisy measure of its own state. In particular, the measure of $x_i$ as acquired by vehicle $i$th is:

$$x_{i,n} = x_i + \phi_i,$$

(5)

where $\phi_i \in \mathbb{R}^n$ is the additive noise such as

$$\|\phi_i\| \leq \gamma \quad \forall \quad i = 1, 2, \ldots, N,$$

(6)

where $\gamma \in \mathbb{R}$ is a positive scalar. The collective noisy measure of the system state is

$$\hat{x}_n = x + \phi,$$

(7)

where $\phi = [\phi_1^T \ldots \phi_N^T]^T \in \mathbb{R}^{Nn}$ is the collective noise.

Exchange of information among agents within the network is represented using a graph $G(\mathcal{E}, \mathcal{V})$ characterized by its topology [12], which consists of a set of vertices (nodes) $\mathcal{V} = \{1, 2, \ldots, N\}$ indexed by the team members and a set of edges (arcs) $\mathcal{E} = \mathcal{V} \times \mathcal{V}$. The graph $G$ is called undirected if the pairs of vertices are unordered, i.e., $(i, j) \in \mathcal{E} \Rightarrow (j, i) \in \mathcal{E}$, otherwise, it is called directed. Two vertices $(i, j)$ are connected if there exists a path between these vertices, and a directed graph is strongly connected, if there is a direct path between every distinct pair of vertices $(i, j)$. An undirected graph is called connected if there is an undirected path between every pair of distinct nodes. A vertex $V_i$ of a directed graph is balanced if the in-degree is equal to the out-degree; a directed graph is balanced if all its vertices are balanced.

It is assumed that the $i$th agent only has access to the state of agents that belong to its communication set $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$, and it does not know the topology of the overall communication graph. The communication topology is characterized by the $(N \times N)$ adjacency matrix $A = \{a_{ij}\}$ given by $a_{ij} = 1$ if there exists an arc from vertex $j$ to vertex $i$, otherwise $a_{ij} = 0$; and the $(N \times N)$ Laplacian matrix defined as $L = \{l_{ij}\}$, such that $l_{ii} = \sum_{j=1,j \neq i}^N a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. The Laplacian matrix exhibits at least a zero eigenvalue with the $N \times 1$ vector of all ones, $1_N$, as the corresponding right eigenvector. Hence, $\text{rank}(L) \leq N - 1$ and $L1_N = 0_N$, where $0_N$ is the $(N \times 1)$ null vector.

For a balanced directed graph, $1_N$ is also a left eigenvector of $L$, i.e., $1_N^T L = 0_N^T$. If the graph is strongly connected $\text{rank}(L) = N - 1$. If the graph is undirected, the Laplacian is symmetric and positive semi-definite; moreover, for a connected graph, $L$ has a single zero eigenvalue. With the corresponding eigenvector $1_N$.

III. STATE OBSERVER

Let $\Gamma_i$ be the $(n \times Nn)$ matrix

$$\Gamma_i = \{O_n \ldots I_n \ldots O_n\},$$

(8)

$i$th node

and $\Pi_i$ be the $(Nn \times Nn)$ matrix $\Pi_i = \Gamma_i^T \Gamma_i$. The following equality holds $\sum_{i=1}^N \Pi_i = I_{Nn}$.

The estimate of the collective state is computed by the $i$th agent $(i = 1, \ldots, N)$ via the observer

$$i\dot{x} = iF(t, i\dot{x}, i\dot{u}) + k_o \left( \sum_{j \in \mathcal{N}_i} (i\dot{x} - i\dot{x}) + \Pi_i (x_n - i\dot{x}) \right),$$

(9)

where $iF(i\dot{x}) = [f(t, i\dot{x}_1, i\dot{u}_1)^T \ldots f(t, i\dot{x}_N, i\dot{u}_N)^T]^T \in \mathbb{R}^{Nn}$, $k_o > 0$ is a scalar gain to be properly selected and

$$i\dot{u}(t, i\dot{x}) = \begin{bmatrix} u_1(t, i\dot{x}) \\ u_2(t, i\dot{x}) \\ \vdots \\ u_N(t, i\dot{x}) \end{bmatrix} \in \mathbb{R}^{Nn}$$

(10)

represents the estimate of the collective input available to the $i$th agent. The input $i\dot{u}(t, i\dot{x})$ is the feedback control law depending on the state of the overall system and whose exact expression is not important at this stage. The only required property is that the control law is a Lipschitz function of the team state:

$$\|u_i(t, y) - u_i(t, x)\| \leq L_u \|y - x\|, \quad i = 1, 2, \ldots, N$$

(11)

\forall x, y \in \mathbb{R}^n$ and where $L_u > 0$ is the Lipschitz constant.

Remark 3.1: The observer (9) requires each agent only uses local information since $\Pi_i$ selects only the $i$th component of the collective state $x$, i.e., the agent’s own state. In addition, exchange of the neighbors estimates is required.
For the sake of notation compactness, the state estimates can be stacked into the vector, \( \hat{x}^* = [\hat{x}^T \ldots \hat{x}_N^T]^T \in \mathbb{R}^{Nn} \), thus, a stacked vector of estimation errors can be defined as well

\[
\tilde{x}^* = \begin{bmatrix}
\hat{x}^* - \hat{x}^* \\
\frac{1}{2} \hat{x}^*_x \\
\vdots \\
\frac{1}{2} \hat{x}_N^* \\
\end{bmatrix} = 1_N \otimes \hat{x}^* - \hat{x}^*,
\]

where the symbol \( \otimes \) represents the Kronecker product.

The collective estimation dynamics is given by

\[
\dot{\tilde{x}}^* = \begin{bmatrix}
1 \hat{F}(t, \hat{x}^*, \hat{u}^*) - k_o (L \otimes I_N) \hat{x}^* + k_o \Pi^e \hat{x}^* + k_o \Pi^e \phi^*, \\
\end{bmatrix}
\]

where \( \phi^* = 1_N \otimes \phi \),

\[
\hat{F}(t, \hat{x}^*, \hat{u}^*) = \begin{bmatrix}
1 \hat{F}(t, \hat{x}^*, \hat{u}^*) \\
2 \hat{F}(t, \hat{x}^*, \hat{u}^*) \\
\vdots \\
N \hat{F}(t, \hat{x}_N^*, \hat{u}_N^*)
\end{bmatrix},
\]

and

\[
\hat{u}^*(t, \hat{x}^*) = \begin{bmatrix}
\hat{u}(t, \hat{x}^*) \\
\vdots \\
\hat{u}_N(t, \hat{x}_N^*)
\end{bmatrix} \in \mathbb{R}^{Nn}.
\]

By noticing that \((L \otimes I_N)(1_N \otimes x) = L1_N \otimes x\) and being \(L1_N = 0_N\), the estimation error dynamics can be derived from (3) and (12) as

\[
\tilde{x}^* = \hat{F}^* - k_o \hat{L}^* \tilde{x}^* + k_o \Pi^e \phi^*,
\]

where \( \hat{F}^* = 1_N \otimes F - \hat{F}^* \), \( \hat{L}^* = (L \otimes I_N + \Pi^e) \).

With regards to matrix \(-\hat{L}^*\), in [1] it was shown that it has all its eigenvalues in the left-half plane in the case of positive definite for connected undirected graphs, as well as for directed balanced and strongly connected topologies.

**IV. OBSERVER STABILITY**

Convergence of the observer scheme is carried out in the case of directed balanced strongly connected topologies.

The candidate Lyapunov function is related to the collective state estimation error

\[
V_o = \frac{1}{2} \tilde{x}^T P \tilde{x}^*,
\]

where \( P \in \mathbb{R}^{Nn \times Nn} \) is a symmetric positive definite matrix. Function \( V_o \) satisfies the following inequality

\[
\lambda_{P_m} \| \tilde{x}^* \|^2 \leq V_o \leq \lambda_{P_m} \| \tilde{x}^* \|^2,
\]

where \( \lambda_{P_m} \) and \( \lambda_{P_m} \) denote, respectively, the smallest and the largest eigenvalue of \( P_m \). The time derivative of \( V_o \) along the system’s trajectories is given by

\[
\dot{V}_o = -k_o \tilde{x}^T (PL^{-T} + \hat{L}^* P) \tilde{x}^* + 2 \tilde{x}^T P (\hat{F}^* + k_o \Pi^e \phi^*).
\]

As matrix \(-\hat{L}^*\) is Hurwitz, given a positive definite matrix \( Q \in \mathbb{R}^{Nn \times Nn} \), it always exists \( P \) such as

\[
P L^{-T} + \hat{L}^* P = Q.
\]

Hence, \( V_o \) can be upper bounded as follows

\[
\dot{V}_o \leq -k_o \lambda_{Q_m} \| \tilde{x}^* \|^2 + \| \tilde{x}^* \|^T \lambda_{P_m} \left( \| \hat{F}^* \| + \| \hat{u}^* \| + k_o \| \Pi^e \phi^* \| \right),
\]

where \( \lambda_{Q_m} \) denotes the smallest eigenvalue of \( Q \). In view of eq. (2) and (10), it holds:

\[
\| \hat{F}^* \| \leq \sum_{i=1}^{N} \sum_{j=1}^{N} |f(t, x_j, u_j(t, \hat{x})) - f(t, \hat{x}_j, u_j(t, \hat{x}))|
\]

\[
\leq \sum_{i=1}^{N} \sum_{j=1}^{N} (L_{f,x} \| x_j - \hat{x}_j \| + L_{f,u} \| u_j(t, \hat{x}) - u_j(t, \hat{x}) \|)
\]

\[
\leq \sum_{i=1}^{N} \sum_{j=1}^{N} (L_{f,x} \| \hat{x} \| + L_{f,u} \| \hat{x} \| \| \hat{x} - \hat{x} \|)
\]

\[
\leq \sum_{i=1}^{N} \sum_{j=1}^{N} (L_{f,x} \| \hat{x} \| + L_{f,u} \| \hat{x} \| \| \hat{x} - \hat{x} \|)
\]

\[
\leq \sum_{i=1}^{N} \sum_{j=1}^{N} (L_{f,x} \| \hat{x} \|^2 + 2L_{f,u} \| \hat{x} \|)
\]

\[
\leq N^2 (L_{f,x} + 2L_{f,u} \| \hat{x} \|).
\]

Based on (6), it is also

\[
\| \Pi^e \phi^* \| = \| \phi \| \leq N\gamma.
\]

By combining eqs. (19)-(21), it is

\[
\dot{V}_o \leq - \left( k_o \lambda_{Q_m} - N^2 \lambda_{P_m} (L_{f,x} + 2L_{f,u}) \right) \| \tilde{x}^* \|^2 + k_o N \lambda_{P_m} \| \tilde{x}^* \|.
\]

Let us choose the gain \( k_o \) such as \( k_o > N^2 \lambda_{P_m} (L_{f,x} + 2L_{f,u}) \). Based on (17), it is clear that \( \tilde{x}^* = 0_{N^2 n} \) is an exponentially stable equilibrium point of the nominal system obtained setting \( \phi^* = 0_{N^2 n} \) (i.e., no measurement noise is present). As showed in [15] (Lemma 9.2, pp.347), the exponential stability of the origin of the nominal system ensures that the solutions of the perturbed system are globally uniformly ultimately bounded provided that the non-vanishing disturbance \( \phi \) is bounded.

**V. CASE STUDY**

In the previous section, it was shown that it is possible for each vehicle to estimate the overall state of the system, \( x \), provided that condition (2) holds. In this section, a possible example is provided.
Let us consider for each vehicle a double integrator dynamics

$$\dot{x}_i = \begin{bmatrix} \dot{p}_i \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} \frac{u_i}{2} \\ \frac{u_i}{2} \end{bmatrix} u_i,$$  \hspace{1cm} (23)

where $p_i$ and $v_i$ are respectively the position and the velocity of the $i$th vehicle. Clearly, the system in eq. (23) is of type (1) and respects condition (2). The choice of a linear dynamics was made in order to short the following convergence proof.

The objective is the decentralized control of centroid and formation of a team of $N$ vehicles. The centroid of the system is

$$\sigma_1(x) = \frac{1}{N} \sum_{i=1}^{N} p_i = \frac{1}{N} \sum_{i=1}^{N} \begin{bmatrix} I_n \otimes [0 \ldots 0] \end{bmatrix} x_i = J_1 S_p x,$$

where $J_1 \in \mathbb{R}^{n \times Nn}$ is the Jacobian of the task, $S_p = I_N \otimes [0 \ldots 0]$ selects the positional components of all the vehicles from the collective state $x = [x_1^T \ldots x_N^T]^T \in \mathbb{R}^{Nn}$ and, finally,

$$J_1 = \frac{1}{N} \left(I_N \otimes I_n \right),$$ \hspace{1cm} (25)

with $I_n$ the $(n \times n)$ identity matrix.

The formation of the system is expressed by means of relative displacement between the agents:

$$\sigma_2(x) = \left[ (p_2-p_1)^T (p_3-p_2)^T \ldots (p_N-p_{N-1})^T \right]^T = J_2 S_v x,$$

where $J_2 \in \mathbb{R}^{(N-1)n \times Nn}$ is the Jacobian of the task

$$J_2 = \begin{bmatrix} -I_n & I_n & O_n & \ldots & O_n & O_n \\ O_n & -I_n & I_n & \ldots & O_n & O_n \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ O_n & O_n & O_n & \ldots & -I_n & I_n \end{bmatrix},$$ \hspace{1cm} (27)

where $O_n$ is the $(n \times n)$ null matrix. It is worth combining both the tasks in one single vector $\sigma$

$$\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} x = JS_p x, \ \ddot{\sigma} = JS_v x,$$ \hspace{1cm} (28)

where $S_p = I_N \otimes [0 \ldots 0]$ selects the velocity components from the state vector $x$. Moreover, it can be easily shown that matrix $J \in \mathbb{R}^{Nn \times Nn}$ is non singular and, then, invertible.

Given a desired value for the task function $\sigma$, $\sigma_d(t)$, the objective is to design the control input $u_i(t, \dot{x}_i)$, $\forall i = 1, 2, \ldots, N$, such that $\dot{\sigma}(t) = \sigma_d(t) - \sigma(t)$ converges asymptotically to the origin.

A. Control law

Let us consider the desired task trajectory $\sigma_d$ and the following control law of the $i$th vehicle:

$$u_i(t, \dot{x}_i) = \Gamma_i J_i^{-1}(\dot{\sigma}_d + K_D(\dot{\sigma}_d - \dot{\sigma}(\dot{x}_i)) + K_P(\sigma_d - \sigma(\dot{x}_i))),$$ \hspace{1cm} (29)

where $\Gamma_i$ was previously defined in eq. (8), and with $K_p, K_d \in \mathbb{R}^{Nn \times Nn}$ diagonal matrix with positive elements on the main diagonal.

Let us consider the dynamics of the task $\sigma$:

$$\dot{\sigma}(t) = J S_p \dot{x} = J S_v \dot{x} = J u,$$ \hspace{1cm} (30)

where the property $S_v \dot{x} = S_v \dot{x}$ was exploited and the fact that, based on eq. (23), $S_v \dot{x}$ represents the collective input $u$ of the system as defined in eq. (4).

The control law (29) can also be rewritten as:

$$u_i(t, \dot{x}_i) = \Gamma_i J_i^{-1}(\dot{\sigma}_d + K_D(\dot{\sigma}_d - JS_v \dot{x}) + K_P(\sigma_d - JS_v \dot{x}))$$

$$= \Gamma_i J_i^{-1}(\dot{\sigma}_d + K_D(\dot{\sigma}_d - JS_v \dot{x}) + K_P(\sigma_d - JS_v \dot{x}))$$

$$= \Gamma_i J_i^{-1}(K_pSJ_p^T \dot{x} + K_dJS_d \dot{x})$$

$$= \Gamma_i J_i^{-1}(K_pSJ_p^T \dot{x} + K_dJS_d \dot{x})$$

$$+ K_i \dot{x}.$$ \hspace{1cm} (31)

where $K_i = \Gamma_i J_i^{-1} \left[ K_pSJ_p \ K_dJS_d \right]$. By considering eqs. (4), (30), (31), and as $[T^T \ T_1^T \ \ldots \ T_N^T]^T a = a$ for any vector $a$ of proper dimensions, the following holds:

$$\dot{\sigma}(t) = \dot{\sigma}_d + K_D(\dot{\sigma}_d - \dot{\sigma}(\dot{x})) + K_P(\sigma_d - \sigma(\dot{x}))$$

$$+ [K_1 \ K_2 \ \ldots \ K_N] \dot{x}.$$ \hspace{1cm} (32)

By defining $\dot{\sigma} = \sigma_d - \sigma$, eq. (32) can be rewritten as

$$\ddot{\sigma} + K_D \dot{\sigma} + K_P \sigma = -K \dot{x},$$ \hspace{1cm} (33)

with $K = [K_1 \ K_2 \ \ldots \ K_N]$. System (33) represents an exponentially stable system perturbed by a bounded input $-K \dot{x}$ (see Section IV), then, $\dot{\sigma}$ is bounded as well. It is worth remarking that in absence of measurement noise $\phi$, $\dot{x}$ asymptotically reaches the origin and the same happens for $\dot{\sigma}$, as $\dot{x}$ would represent an exponentially vanishing perturbation.

VI. NUMERICAL SIMULATIONS

In order to prove the effectiveness of the proposed decentralized approach, results of two different numerical simulations implemented in Matlab environment are reported. Specifically, a team of 5 agents is considered as 2D ($n = 2$) multi-robot system characterized by the communication network topology shown in Figure 1. The team centroid is commanded to track a desired time-varying reference by moving along a desired spiral trajectory, while two different assigned formations task are addressed. The gains $K_p, K_d$ and $k_o$ in eq. (29) and in eq. (9) have been set to $3.0I$, $3.0I$ and $3.0$ respectively.

In the first simulation, the assigned task formation is a circular shape formation. Figure 2 shows the desired path of the team together with the desired formation at the beginning, intermediate and the end of the mission. In Figure 3 the dotted lines show the real paths of the robots $x_i$ during the mission, while the solid lines represent the paths of all the
robots as estimated by one of them. It is possible noticing that the real paths almost coincide with the desired ones. Figure 4 shows the time history of the estimation error norm $\|\tilde{x}^*\|$, as well as the norm of the task function errors $\|\tilde{\sigma}_1\|$ and $\|\tilde{\sigma}_2\|$.

While the Figure 5 shows the norms of input values to the robots.

In the second simulation we considerer the task of repositioning the formation of the team of the robots team during the mission; in particular, the robots are expected to change the shape of the team from a circular formation to a linear one. In Figure 6 the dotted lines show the real paths of the robots during the simulation, while the solid lines represent the paths of all robots as estimated by one of them. It is possible noticing that the desired paths is well tracked by the robots as expected. Figure 7 shows the time history of the estimation error norm, as well as the norm of the two task functions errors, while the Figure 8 presents the velocity of the robots during the whole mission.

VII. CONCLUSIONS

In this paper, a decentralized observer for a class of multi-robot system is presented. The observer allows each agent to estimate the collective state of the system by using only local information and it is used, together with a Lipschitz control law, to cooperatively track global tasks, defined in terms of system’s centroid and geometrical formation. The validation is supported by results of numerical simulations with a distributed multi-agent system.
Fig. 6. Paths of the robots measured (dotted lines) and estimated by robot 1 (solid line).

Fig. 7. State and tasks’ estimation errors. (a): observer error $\|\tilde{x}^*\|$. (b): centroid error $\|\tilde{\sigma}_1\|$. (c): relative distance error $\|\tilde{\sigma}_2\|$

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Fig. 8. Control inputs of the robots.