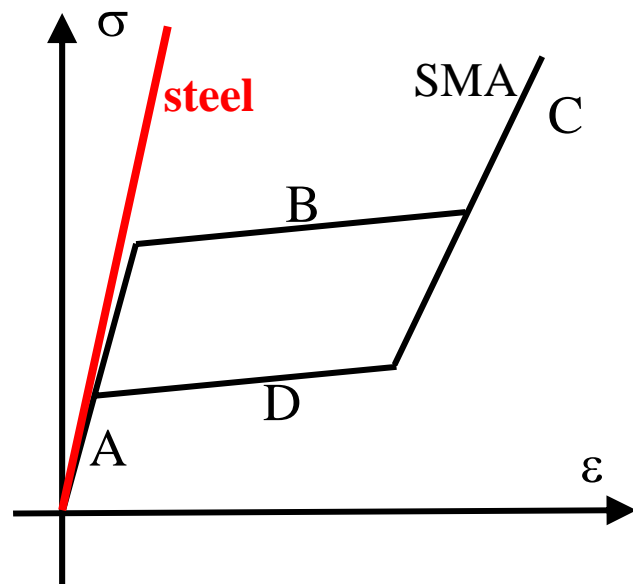
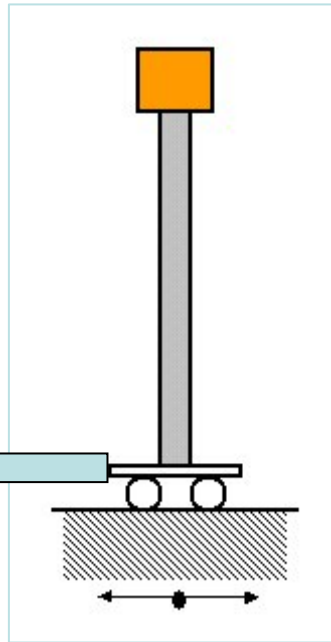


► plateau

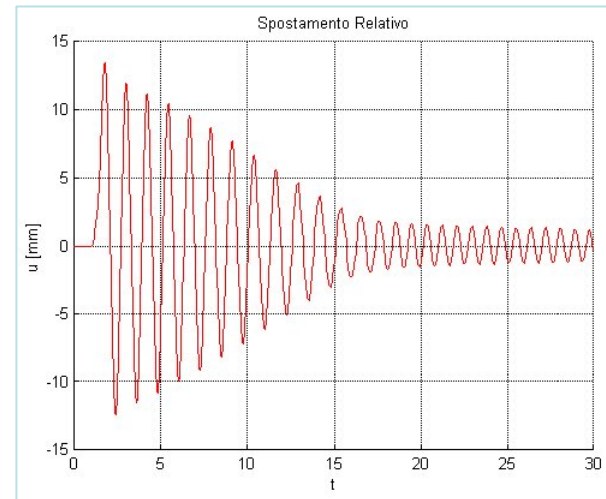
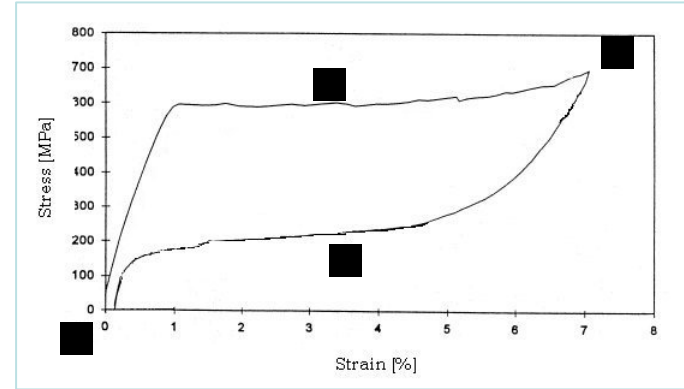
## SMA orthodontic wires

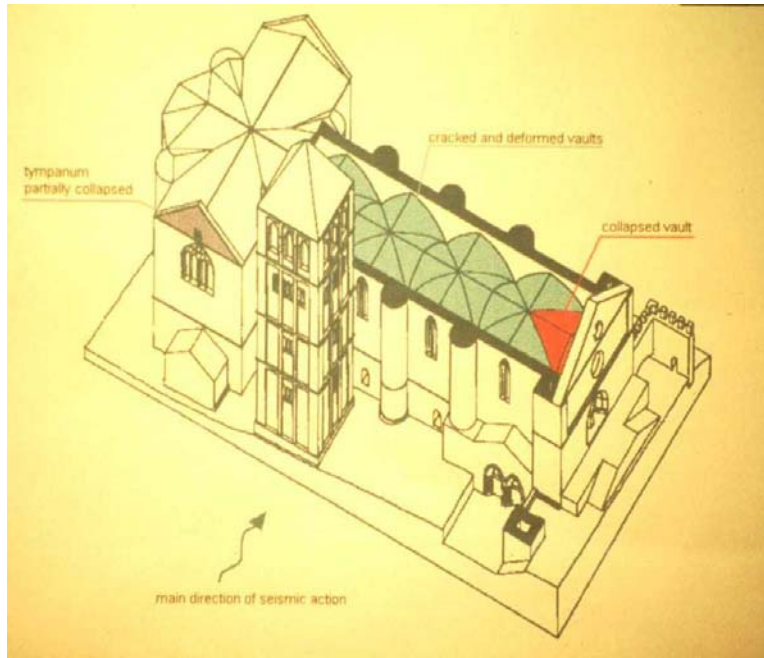


**SMA  
damper**

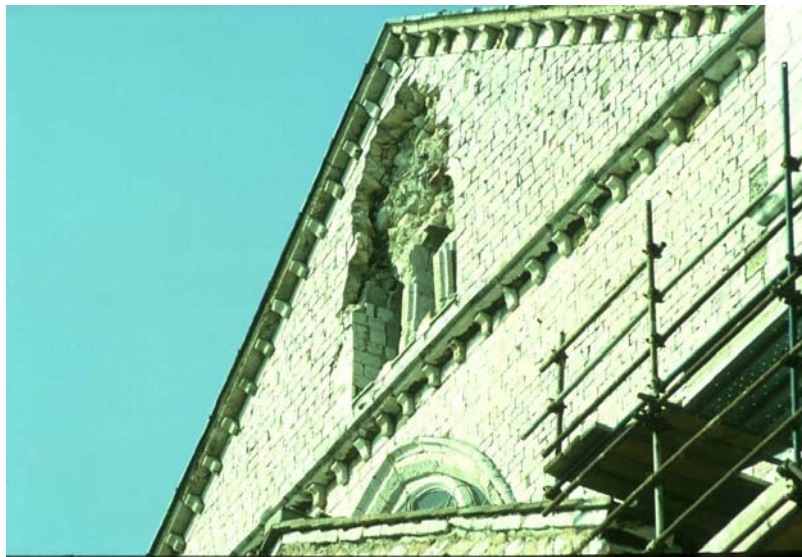


$$m \ddot{x} + c \dot{x} + k x = F_0 \sin(\omega t)$$





# Timpano



26/9/97

7/10/97



In order to reduce the value of the seismic forces applied to the “*timpano*”, the junctions between the “*timpano*” and the roof are made of SMA (constant force and energy dissipation).



# SMA modeling

- **Microscopic Thermodynamics Models**
- **Macroscopic Phenomenological Models**
- **Micromechanics Based Macroscopic Models**

## **Microscopic Thermodynamics Models**

phenomenological thermodynamics to describe an infinitesimal volume in an infinite domain

explanation of the micro-scale behavior, such as nucleation, interface motion and growth of a martensite plate.

extremely helpful for understanding the phenomenon but difficult to apply for engineering applications.

**Abeyaratne and Knowles. *Journal of the Mechanics and Physics of Solids* 1990**

**Ball and James. *Archives of Rational Mechanics and Analysis* 1987**

**Barsch and Krumhansl *Martensite ASM International* 1992**

**Falk *Z. Physik B-Condensed Matter* 1983**



## **Micromechanics Based Macroscopic Models**

thermodynamics to describe the transformation and  
micromechanics to estimate the interaction energy due to the  
transformation in the material

interaction energy: key factor in the transformation mechanism

knowledge of the microstructural evolution is required

assumptions at the microstructure level have been made to  
approximate the interaction energy

**Fischer and Tanaka. *International Journal of Solids and Structures* 1992**

**Raniecki et al. *Arch. Mech.* 1992**

**Sun and Hwang. *Journal of the Mechanics and Physics of Solids* 1993**

**Patoor et al. *Mechanics of Phase Transformations and Shape Memory Alloys* 1994**

# Macroscopic Phenomenological Models

phenomenological thermodynamics curve fitting from experimental data

based on the phase diagram of SMA transformation

martensite volume fraction used as an internal variable

suitable for engineering applications due to their simplicity

accurate models (built on experimental data)

lack of multi-dimensional experimental tests

difficult to extend phenomenological models to 2-D and 3-D cases

three-dimensional macroscopic phenomenological models proposed with some success

Auricchio and Taylor. *Comp Methods Appl. Mech. Engrg.* 1997

Brinson. *J. of Intell. Mat. Syst. and Struct.* 1993

Bekker and Brinson. *Journal of the Mechanics and Physics of Solids* 1997

Ivshin and Pence. *Int. J. Engng. Sci.* 1994

Liang and Rogers. *Journal of Engineering Mathematics* 1992

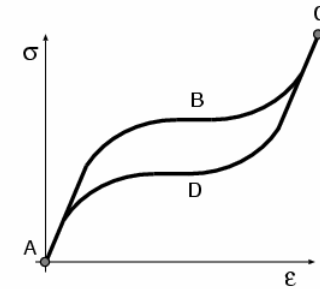
Sato and Tanaka. *Res. Mech.* 1988

Boyd and Lagoudas. *Int. J. Plasticity* 1995

Liang and Rogers. *Journal of Engineering Mathematics* 1992



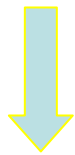
# Superelastic 1D SMA model



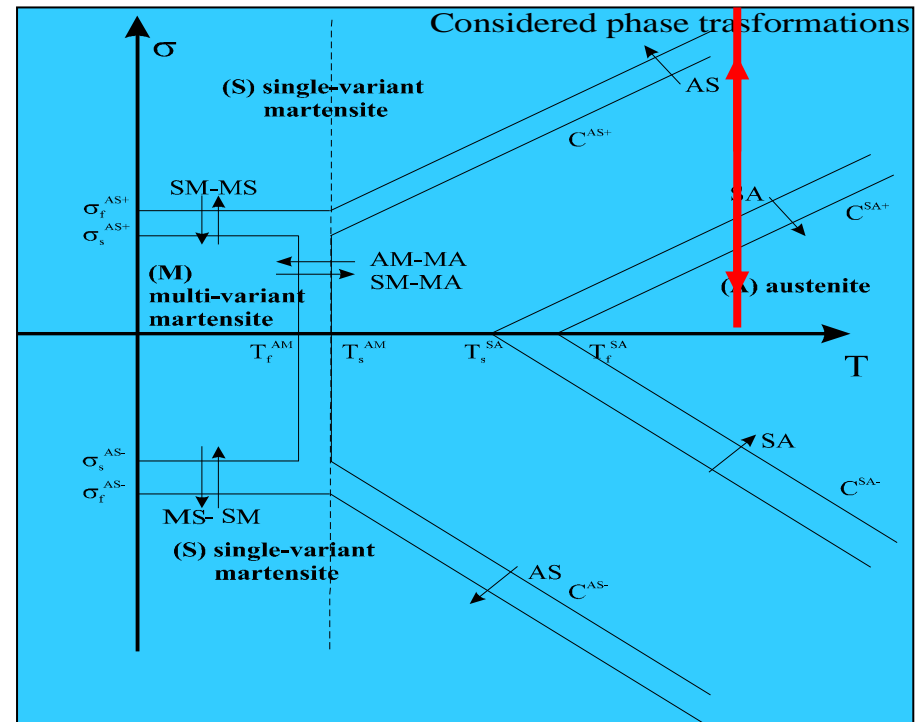
$\xi_A$  austenite volume fraction

$\xi_S$  single variant martensite volume fraction

$$\xi_A + \xi_S = 1$$



$$\xi_S = 1 - \xi_A$$



## Stress-strain relationship

$$\sigma = E(\varepsilon - \xi_S \varepsilon_L^{(\pm)}) \quad \varepsilon_L^{(\pm)} \text{ maximum residual strain}$$

## Evolution of the Young modulus

$$E(\xi) = \frac{E^A E^S}{E^S + \xi(E^A - E^S)}$$

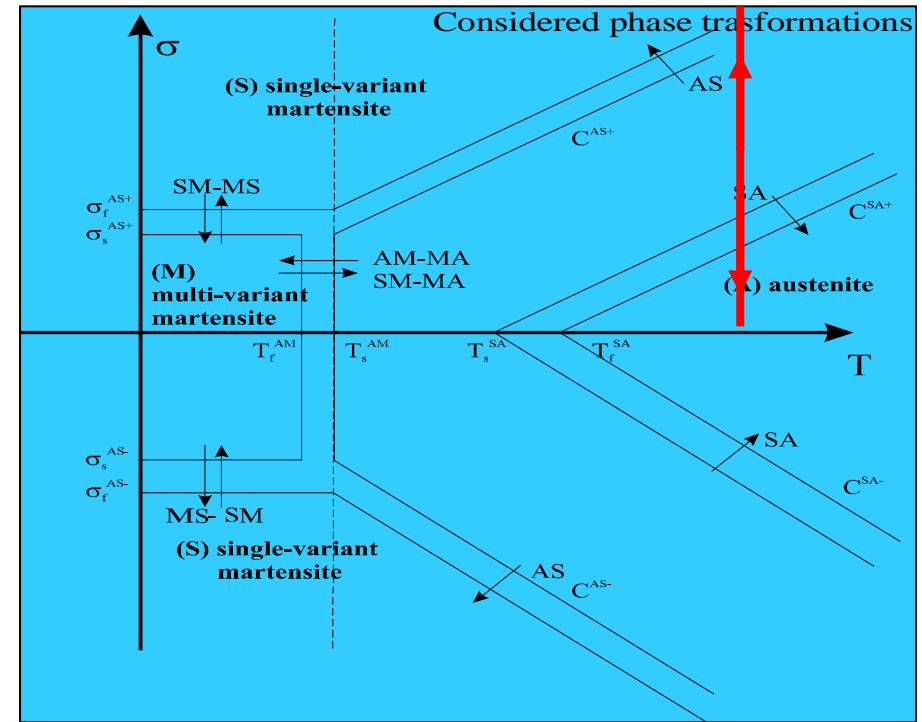
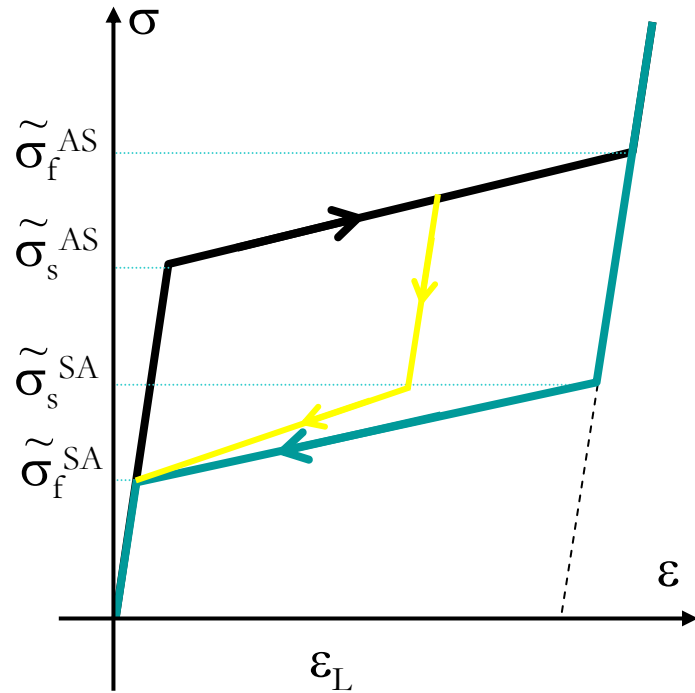
$E^A$  austenite Young modulus

$E^S$  martensite Young modulus



Reuss homogenization

# Evolution of the internal parameter



## austenite - martensite evolution

when  $\sigma_s^{AS(\pm)} \leq |\sigma| \leq \sigma_f^{AS(\pm)}$

$$\dot{\xi}_S = -(1 - \xi_S) \frac{1}{|\sigma| - \sigma_f^{AS(\pm)}} \dot{\sigma}$$

## martensite - austenite evolution

when  $\sigma_s^{SA(\pm)} \leq |\sigma| \leq \sigma_f^{SA(\pm)}$

$$\dot{\xi}_S = \xi_S \frac{1}{|\sigma| - \sigma_f^{SA(\pm)}} \dot{\sigma}$$

## Note

when  $\sigma_s^{AS(\pm)} \leq \sigma \leq \sigma_f^{AS(\pm)}$

$$\dot{\xi}_S = -(1 - \xi_S) \frac{1}{\sigma - \sigma_f^{AS(\pm)}} \dot{\sigma} \quad \rightsquigarrow \quad \frac{\dot{\xi}_S}{(1 - \xi_S)} = -\frac{1}{\sigma - \sigma_f^{AS(\pm)}} \dot{\sigma}$$

$$\int_0^{\xi_{S,t}} \frac{\dot{\xi}_S}{(1 - \xi_S)} = -\int_{\sigma_s^{AS(\pm)}}^{\sigma_t} \frac{1}{\sigma - \sigma_f^{AS(\pm)}} \dot{\sigma}$$

$$-\ln(1 - \xi_{S,t}) = -\ln(\sigma_f^{AS(\pm)} - \sigma_t) + \ln(\sigma_f^{AS(\pm)} - \sigma_s^{AS(\pm)})$$

$$\ln\left(\frac{1}{1 - \xi_{S,t}}\right) = \ln\left(\frac{\sigma_f^{AS(\pm)} - \sigma_s^{AS(\pm)}}{\sigma_f^{AS(\pm)} - \sigma_t}\right) \quad \rightsquigarrow \quad \frac{1}{1 - \xi_{S,t}} = \frac{\sigma_f^{AS(\pm)} - \sigma_s^{AS(\pm)}}{\sigma_f^{AS(\pm)} - \sigma_t}$$

$$\xi_{S,t} = \frac{\sigma_t - \sigma_s^{AS(\pm)}}{\sigma_f^{AS(\pm)} - \sigma_s^{AS(\pm)}}$$

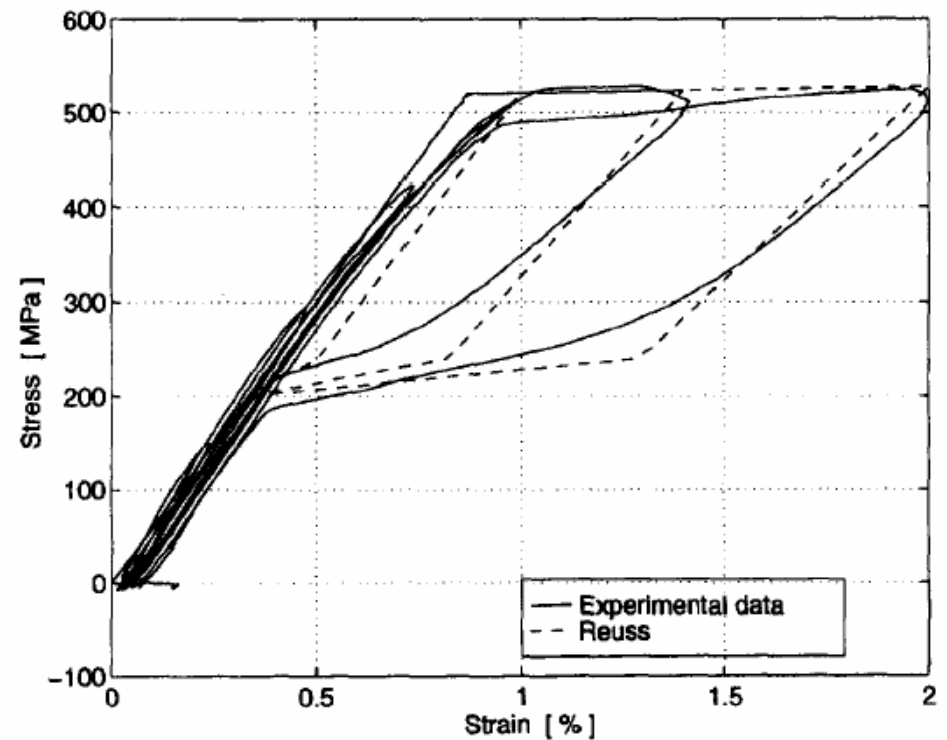
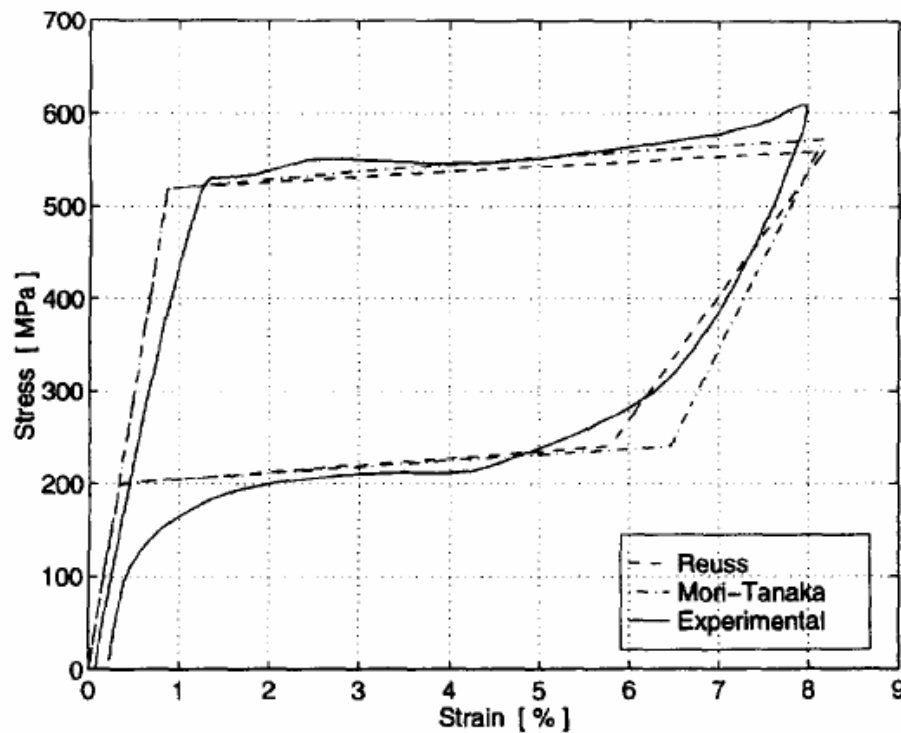
# Results

NDC Ni-Ti alloy provided by Nitinol Device & Components

$$E_A = 60,000 \text{ MPa}, \quad E_S = 7500 \text{ MPa}, \quad \varepsilon_L = 8\%$$

$$\sigma_s^{AS} = 520 \text{ MPa}, \quad \sigma_f^{AS} = 600 \text{ MPa}$$

$$\sigma_s^{SA} = 240 \text{ MPa}, \quad \sigma_f^{SA} = 200 \text{ MPa}$$

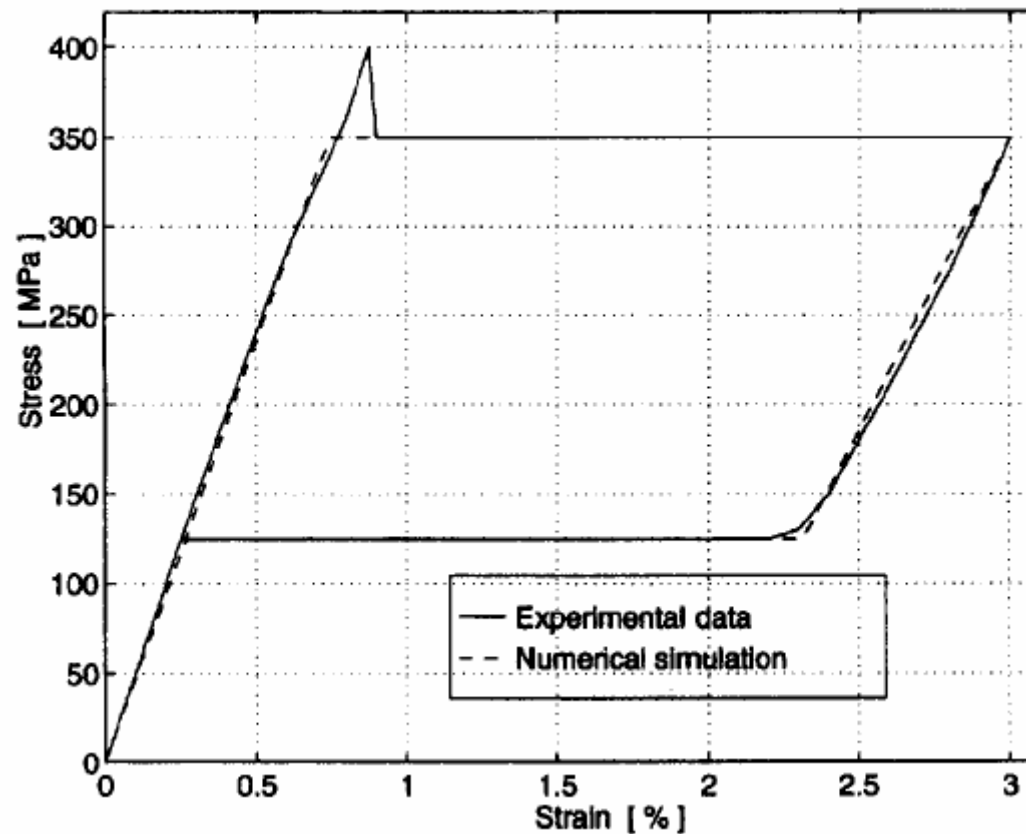


# GAC Ni-Ti alloy provided by GAC International Inc.

$$E_A = 47,000 \text{ MPa}, \quad E_S = 17,000 \text{ MPa}, \quad \varepsilon_L = 8\%$$

$$\sigma_s^{AS} = 350 \text{ MPa}, \quad \sigma_f^{AS} = 350 \text{ MPa}$$

$$\sigma_s^{SA} = 125 \text{ MPa}, \quad \sigma_f^{SA} = 125 \text{ MPa}$$



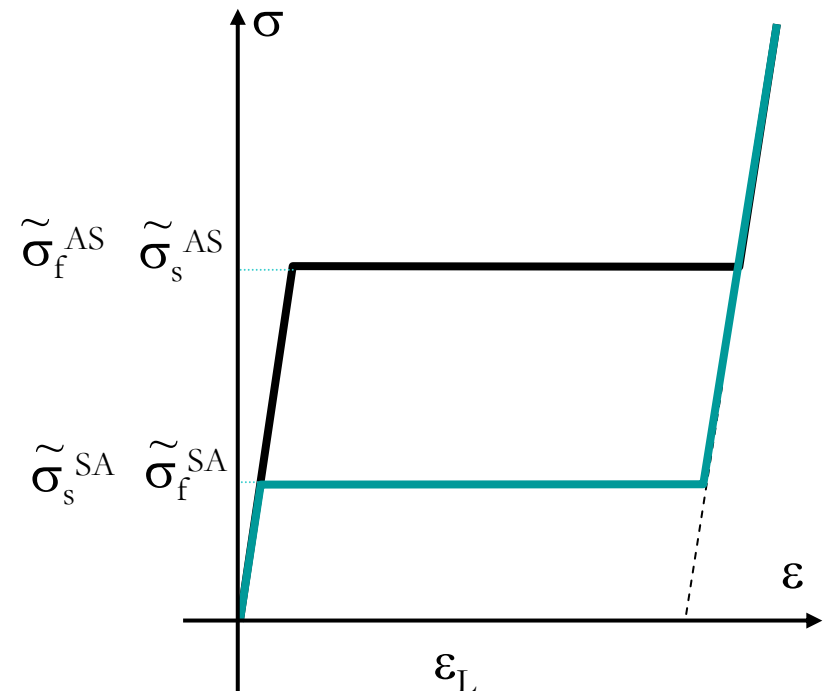


# Beam problem

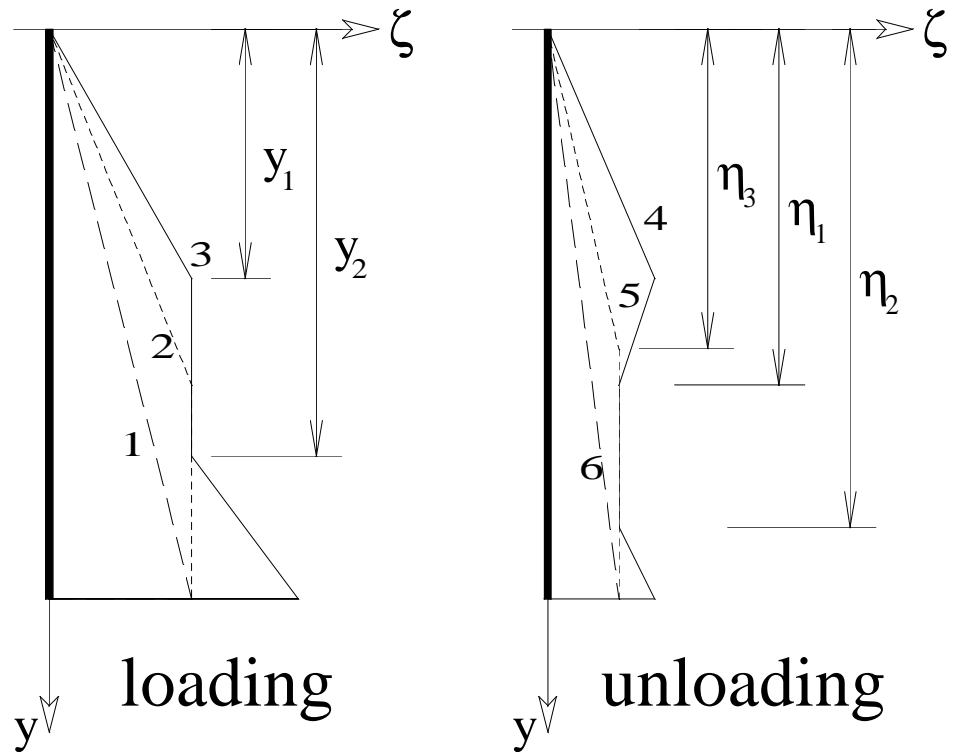
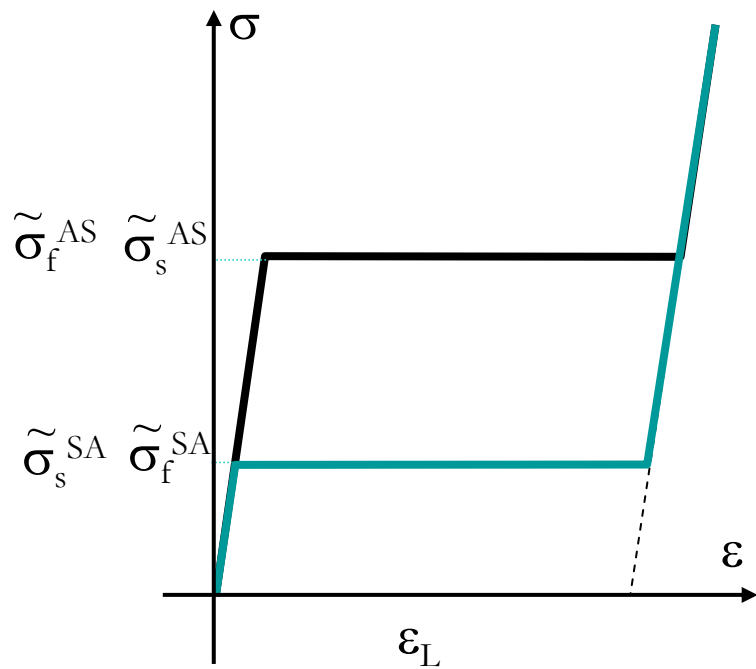
1. *Kinematics*
2. *Constitutive model*
3. *Equilibrium equations*
4. Numerical procedures (fiber model)

## Appendix: An analytical solution

- equal material response in tension and compression,
- equal elastic properties between austenite and martensite,
- equal initial and final values for the phase transformations,
- pure bending loading state.

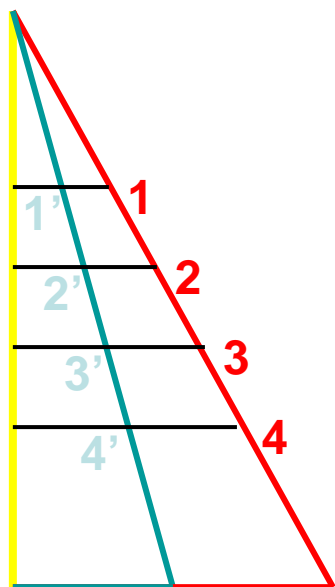


$y_1 :$	$E\varepsilon(y_1) = \sigma^{AS}$	with	$0 \leq y_1 \leq \frac{h}{2}$	rectangular cross-section $b \times h$
$y_2 :$	$E[\varepsilon(y_2) - \varepsilon_L] = \sigma^{AS}$	with	$0 \leq y_2 \leq \frac{h}{2}$	
$\eta_1 :$	$E[\varepsilon(\eta_1) - \varepsilon_p(\eta_1)] = \sigma^{SA} - \sigma^{AS}$	with	$y_{1,p} \leq \eta_1 \leq y_{2,p}$	
$\eta_2 :$	$E[\varepsilon(\eta_2) - \varepsilon_L] = \sigma^{SA}$	with	$y_{2,p} \leq \eta_2 \leq \frac{h}{2}$	
$\eta_3 :$	$E\varepsilon(\eta_3) = \sigma^{SA}$	with	$y_{1,p} \leq \eta_3 \leq \frac{h}{2}$	



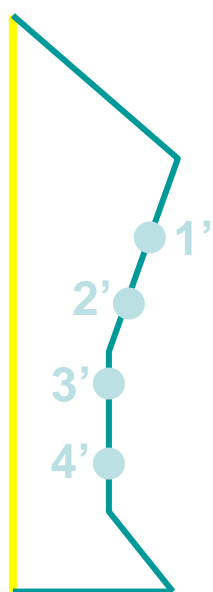
# unloading

section



strain

stress

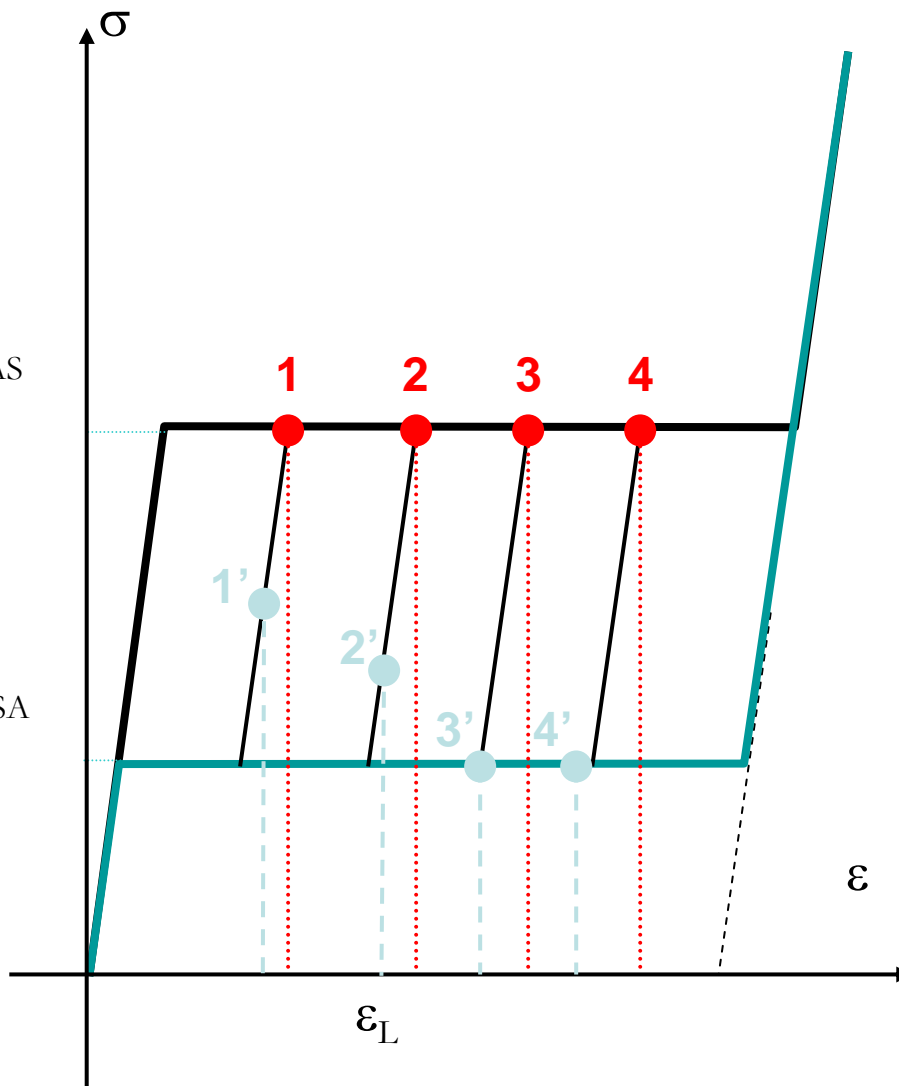


$$\tilde{\sigma}_f^{AS}$$

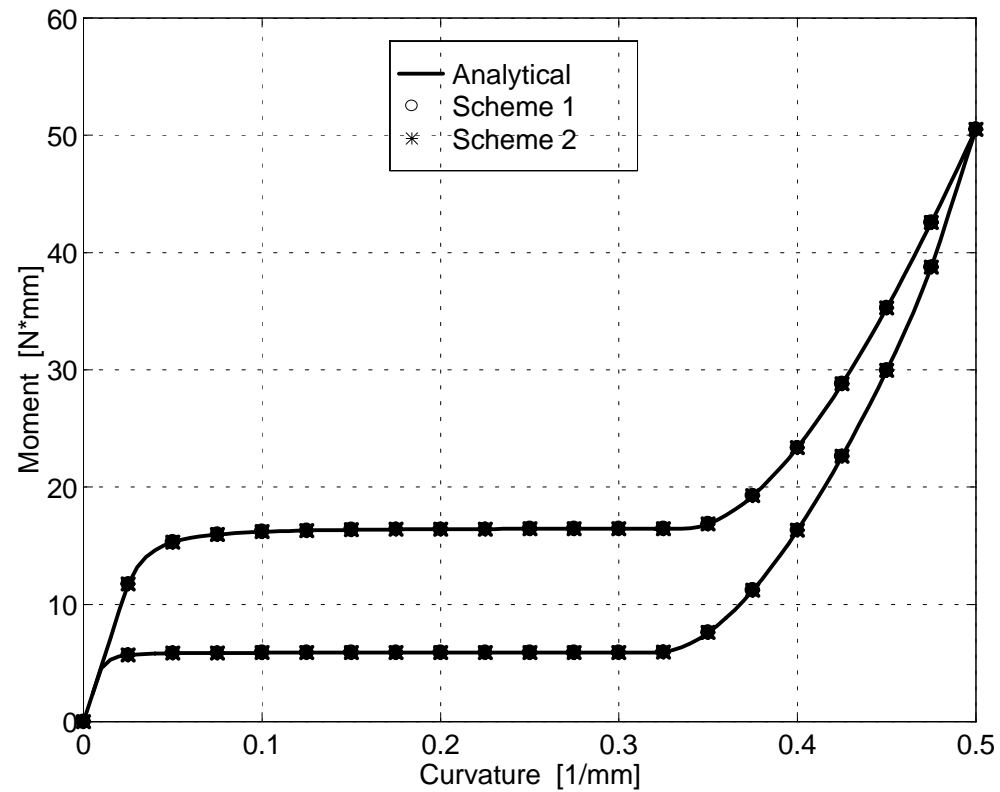
$$\tilde{\sigma}_s^{AS}$$

$$\tilde{\sigma}_s^{SA}$$

$$\tilde{\sigma}_f^{SA}$$



*Comparison between analytical and numerical solutions  
(GAC Ni-Ti alloy)*



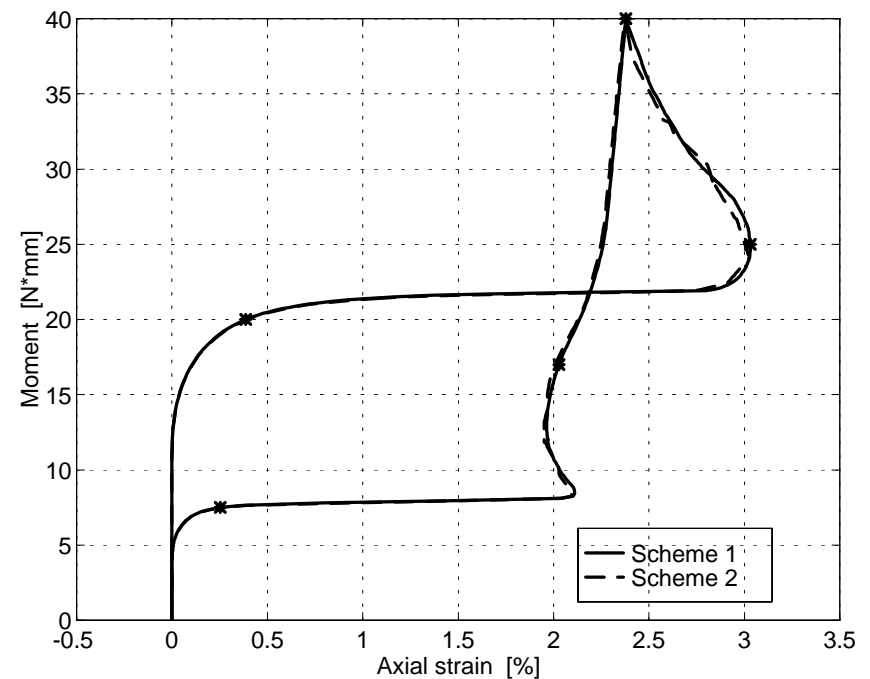
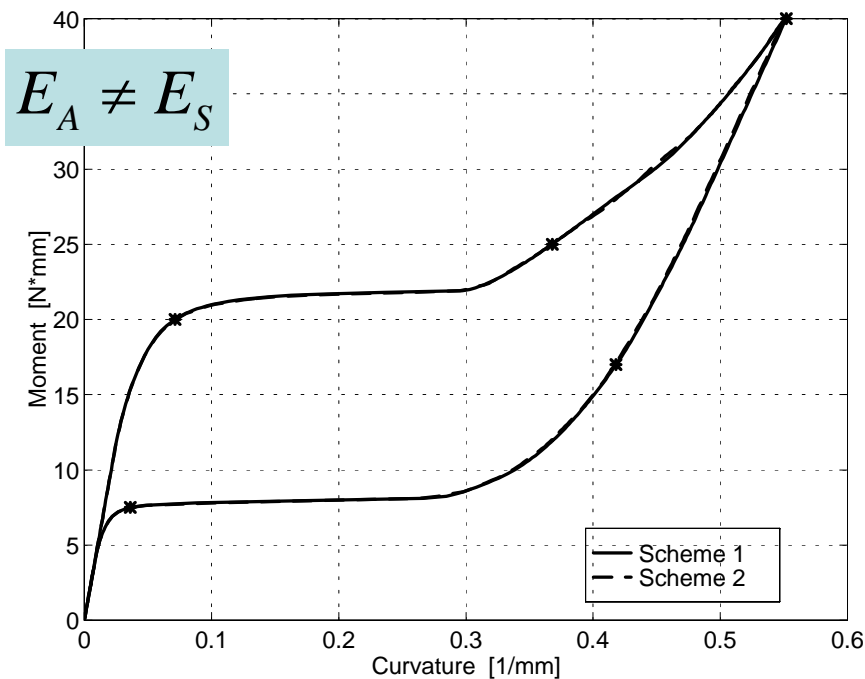
bending moment versus the curvature

# Stress distribution in the cross-section evolution during the loading process

1. no phase transformation, linear elastic material response, neutral axis at zero.
2. phase transformation on the part of the cross-section in traction (stress transformation higher in compression than in tension), then transformation in compression; neutral axis starts to move downward and axial deformation shows up.
3. material restiffening when the phase transformation is completed in traction; progressive upward movement of the neutral axis, reduction of the axial deformation..

**Unloading process** stress pattern more and more complex: combination of the neutral axis movement and the different response in tension and in compression.

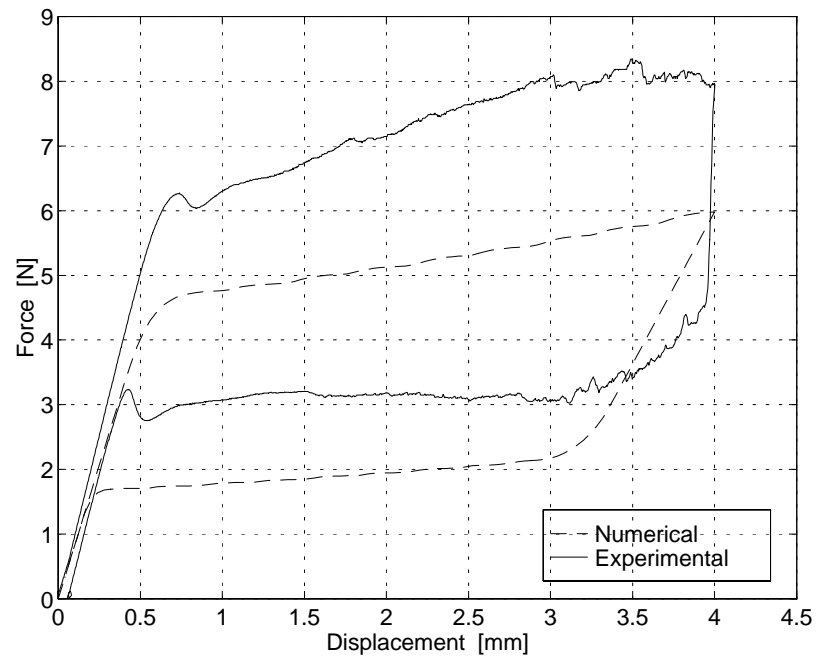
## curvature and axial strain versus the bending moment



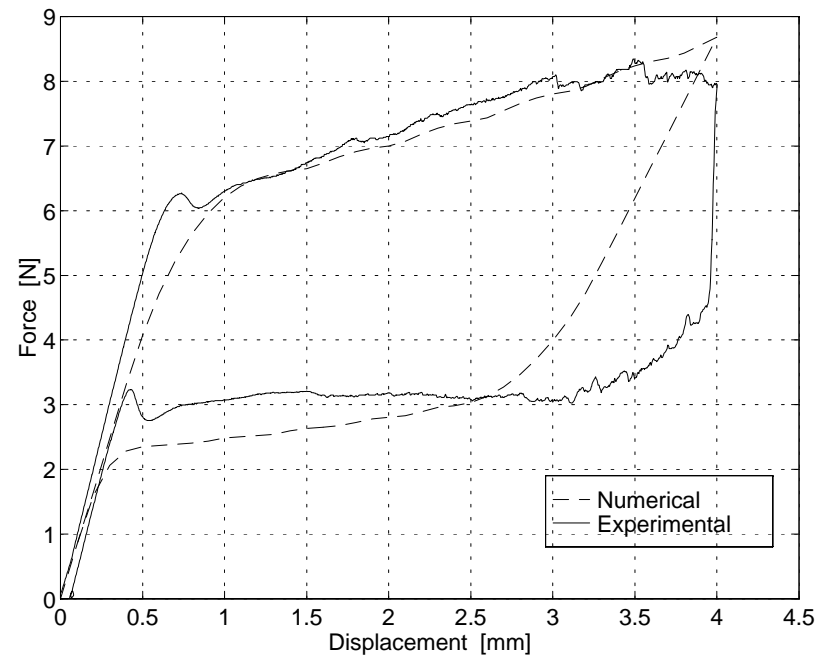
# Three point bending

## GAC wire

equal response in tension and compression



different material response in tension and compression



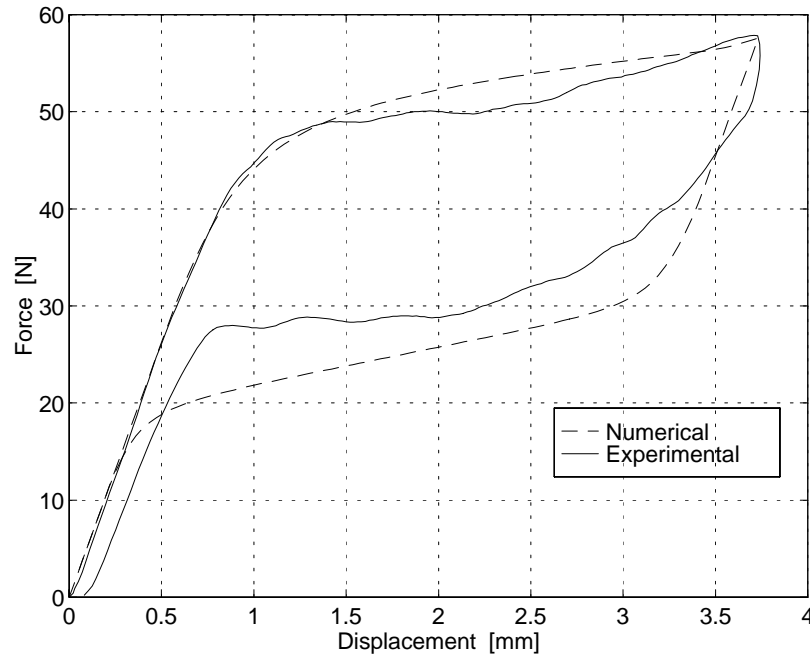


Four point bending

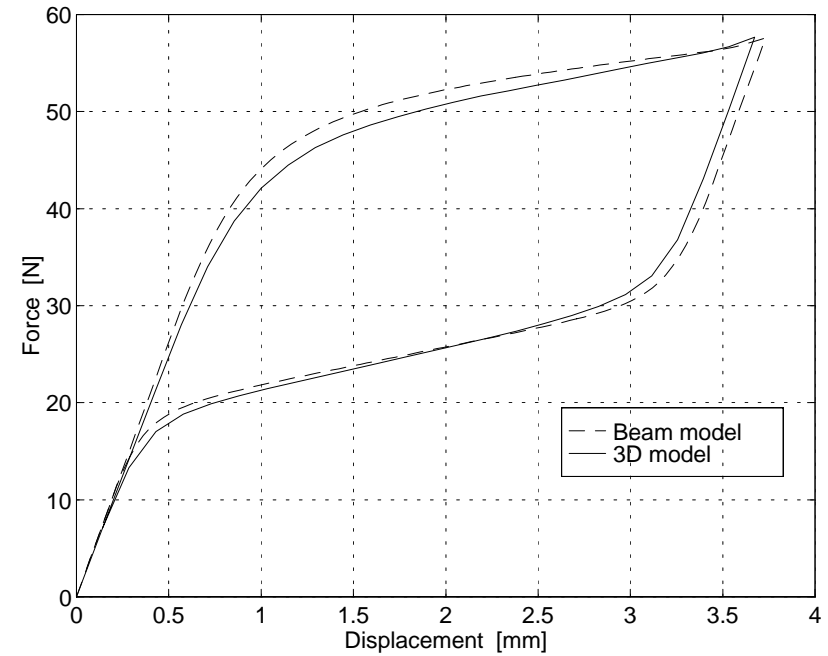
NDC circular wire

**equal properties in traction and in compression and equal elastic moduli between austenite and martensite.**

**comparison between numerical and experimental results**



**comparison between beam and three-dimensional analyses**

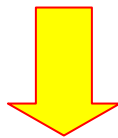


applied force  $F$  versus midspan inflection

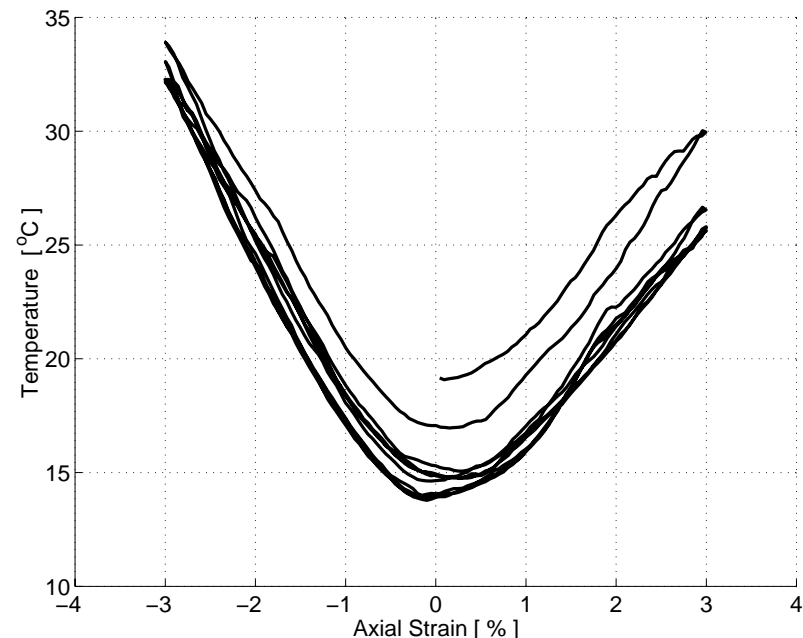
# Thermomechanical modeling of SE-SMA wires

Experimental data reveal:

- material response characterized by an hysteresis, indicating a mechanical dissipation;
- superelastic response associated to significant temperature variations.
- austenite-martensite phase transformations occur in well defined stress-temperature ranges and they can be induced through mechanical loading-unloading patterns, through thermal cooling-heating patterns or through combined mechanical-thermal patterns;



SMA's show strong thermo-mechanical constitutive coupling.



**typical superelastic response in tension and compression [Lim and McDowell (1999)]**

## Single-phase material free-energy [Raniecki – Bruhns (1991)]

$$\psi(\varepsilon^{el}, T) = \psi_0(T) + \psi^{el}(\varepsilon^{el}) - (T - T_0)s^{el}(\varepsilon^{el}, T) + C \left[ (T - T_0) - T \log \frac{T}{T_0} \right]$$

$\psi_0 = u_0 - Ts_0$  free-energy in the natural or reference state ( $T = T_0, \varepsilon^{el} = 0$ )

$u_0$  internal energy and  $s_0$  entropy in the reference state.

$$\psi^{el} = \frac{1}{2} E (\varepsilon^{el})^2$$

free-energy increase due to the elastic strain for  $T = T_0$

$$s = -\frac{\partial \psi}{\partial T} = s_0 + s^{el}(\varepsilon^{el}) + C \log \frac{T}{T_0}$$

entropy

$$s^{el}(\varepsilon^{el}) = \varepsilon^{el} E \alpha$$

entropy increase due to the elastic strain for  $T = T_0$

$\alpha$  thermal expansion factor

$$C = T \frac{\partial s}{\partial T}$$

heat capacity

### final form of the free-energy

$$\psi = u_0 - Ts_0 + \frac{1}{2} E (\varepsilon^{el})^2 - (T - T_0) \varepsilon^{el} E \alpha + C \left[ (T - T_0) - T \log \frac{T}{T_0} \right]$$

## Material undergoing phase transformation

$$\psi_0 = [u_A - Ts_A] - \xi_S [\Delta u - T\Delta s] + \xi_S (1 - \xi_S) [u_{int} - Ts_{int}]$$

$u_A$   $s_A$  internal energy and the entropy of the austenite

$\Delta u$   $\Delta s$  internal energy difference and the entropy difference between the austenite and the martensite

$u_S$  internal energy and the entropy of the martensite

$u_{int} - Ts_{int}$  interaction term

## free-energy form for a material undergoing a solid-solid PT

$$\psi = \frac{1}{2} E (\varepsilon - \varepsilon_L \xi_S s)^2 - (T - T_0) (\varepsilon - \varepsilon_L \xi_S s) E \alpha + C \left[ (T - T_0) - T \log \frac{T}{T_0} \right] + \left[ (u_A - Ts_A) - \xi_S (\Delta u - T\Delta s) \right]$$

stress and entropy

$$\sigma = \frac{\partial \psi}{\partial \varepsilon^{el}} = \frac{\partial \psi}{\partial \varepsilon} = E(\varepsilon - \varepsilon_L \xi_S s) - E\alpha(T - T_0)$$

$$s = -\frac{\partial \psi}{\partial T} = s_A - \xi_S \Delta s + C \log \frac{T}{T_0} + (\varepsilon - \varepsilon_L \xi_S s) E \alpha$$

heat equation

$$C\dot{T} + \text{div } q = b$$

$$b = H_{\text{tmc}} + D_{\text{mec}} \quad \text{heat source}$$

heat production associated to the thermomechanical coupling

$$H_{\text{tmc}} = T \frac{\partial^2 \psi}{\partial T \partial \varepsilon} \dot{\varepsilon} + T \frac{\partial^2 \psi}{\partial T \partial \xi_S} \dot{\xi}_S = T \left[ -E\alpha \dot{\varepsilon} + (\varepsilon_L E\alpha + \Delta s) \dot{\xi}_S \right]$$

heat production associated to dissipative mechanical processes

$$D_{\text{mec}} = \sigma \dot{\varepsilon} - \left( \frac{\partial \psi}{\partial \varepsilon} \dot{\varepsilon} + \frac{\partial \psi}{\partial \xi_S} \dot{\xi}_S \right) = (\varepsilon_L |\sigma| - T\Delta s + \Delta u) \dot{\xi}_S$$