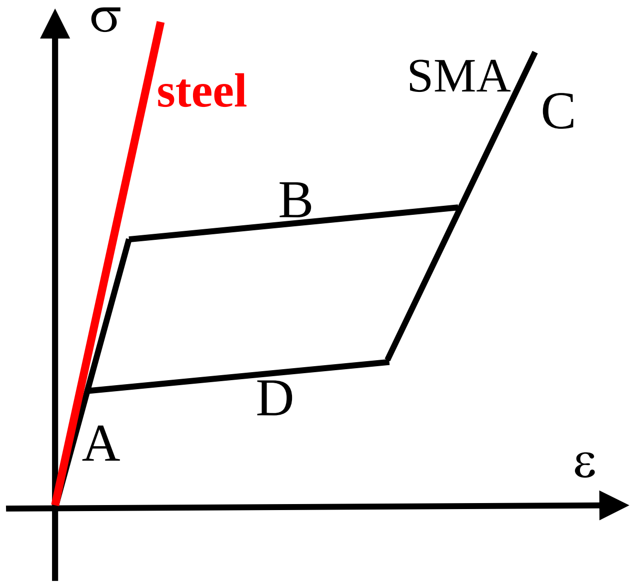
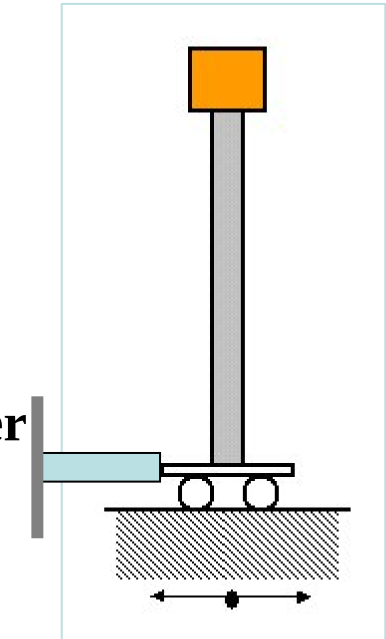
► *plateau*

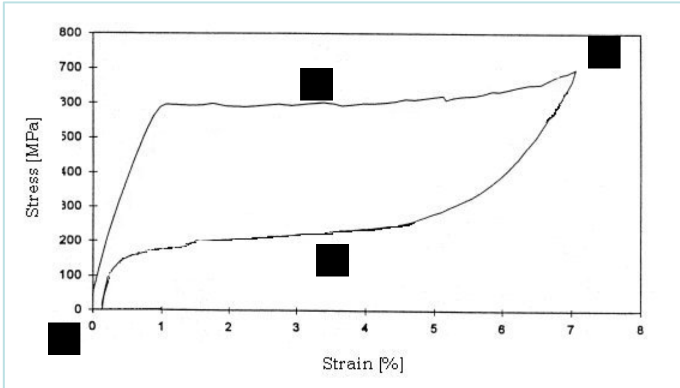
SMA orthodontic wires

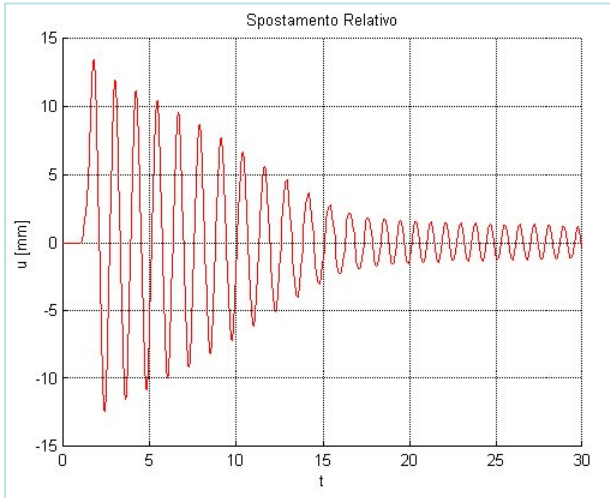


SMA damper

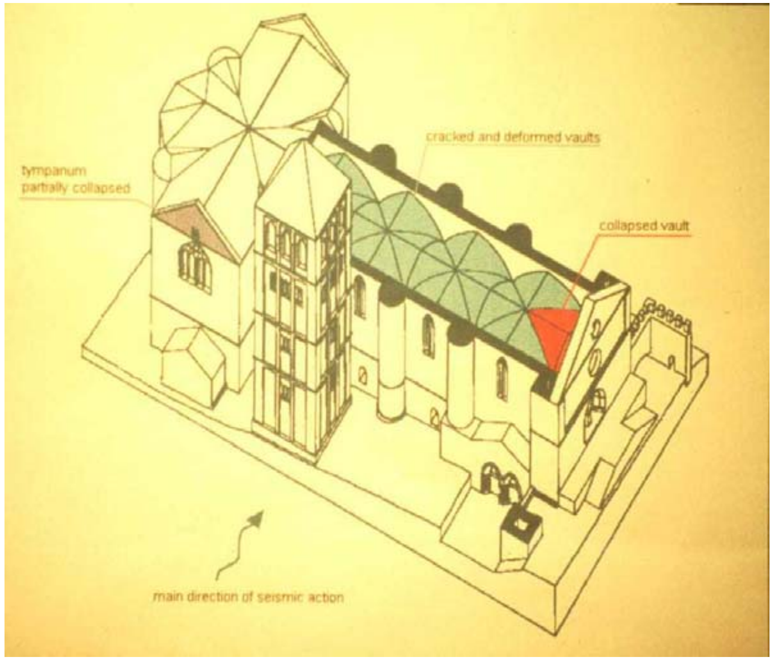


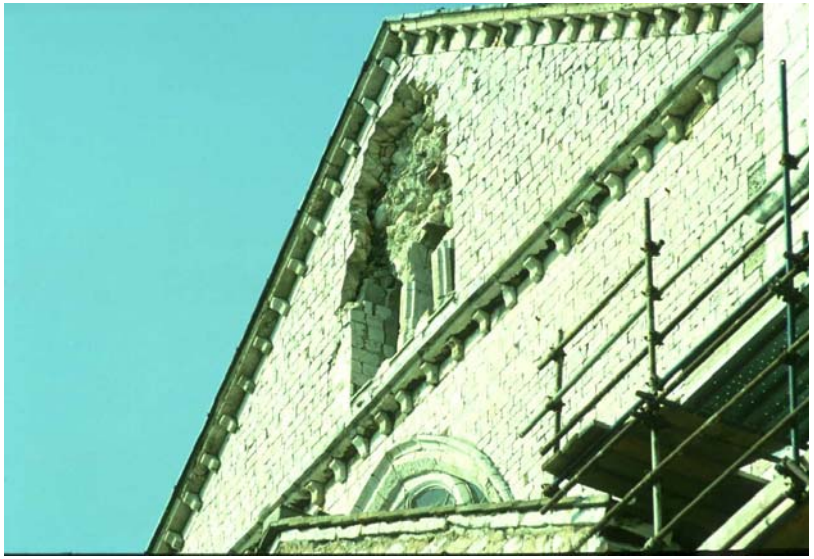
$$m\,\ddot{x} + c\,\dot{x} + k\,x = F_0\,sin(\omega\,t)$$





Timpano





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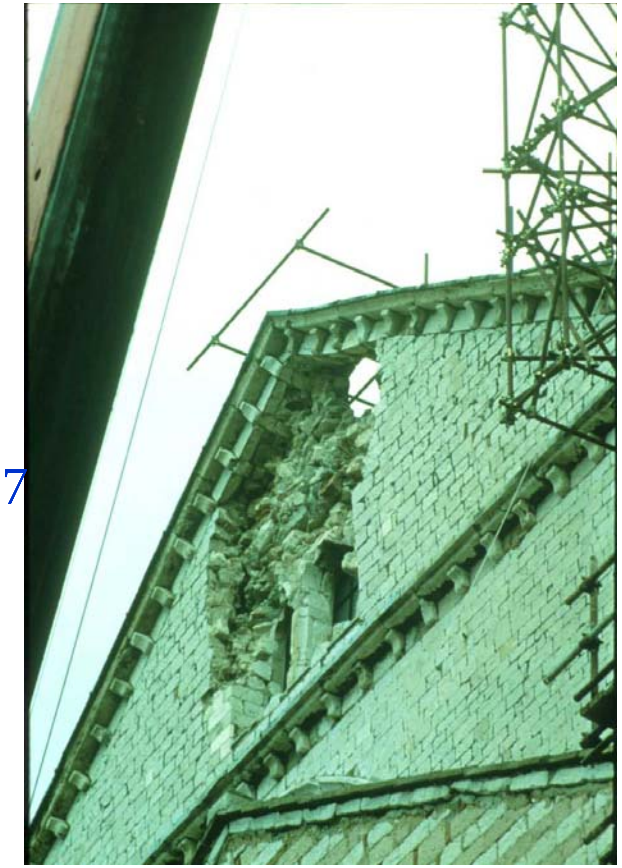


In order to reduce the value of the seismic forces applied to the "*timpano*", the junctions between the "*timpano*" and the roof are made of SMA (constant force and energy dissipation).

SMA modeling

- **Microscopic Thermodynamics Models**
- **Macroscopic Phenomenological Models**
- **Micromechanics Based Macroscopic Models**

Microscopic Thermodynamics Models

phenomomenological thermodynamics to describe an infinitesimal volume in an infinite domain

explanation of the micro-scale behavior, such as nucleation, interface motion and growth of a martensite plate.

extremely helpful for understanding the phenomenon but difficult to apply for engineering applications.

Abeyaratne and Knowles. *Journal of the Mechanics and Physics of Solids* 1990
Ball and James. *Archives of Rational Mechanics and Analysis* 1987
Barschand Krumhansl *Martensite ASM International* 1992
Falk *Z. Physik B-Condensed Matter* 1983

Micromechanics Based Macroscopic Models

thermodynamics to describe the transformation and micromechanics to estimate the interaction energy due to the transformation in the material

interaction energy: key factor in the transformation mechanism

knowledge of the microstructural evolution is required

assumptions at the microstructure level have been made to approximate the interaction energy

Fischer and Tanaka. *International Journal of Solids and Structures* 1992
Raniecki et al. *Arch. Mech*. 1992
Sun and Hwang. *Journal of the Mechanics and Physics of Solids* 1993
Patoor et al. *Mechanics of Phase Transformations and Shape Memory Alloys* 1994

Macroscopic Phenomenological Models

phenomenological thermodynamics curve fitting from experimental data

based on the phase diagram of SMA transformation

martensite volume fraction used as an internal variable

suitable for engineering applications due to their simplicity

accurate models (built on experimental data)

lack of multi-dimensional experimental tests

difficult to extend phenomenological models to 2-D and 3-D cases

three-dimensional macroscopic phenomenological models proposed with some success

Auricchio and Taylor. *Comp Methods Appl. Mech.Engrg*. 1997
Brinson. *J. of Intell. Mat. Syst. and Struct*. 1993

Bekker and Brinson. *Journal of the Mechanics and Physics of Solids* 1997

Ivshin and Pence. *Int. J. Engng. Sci.* 1994

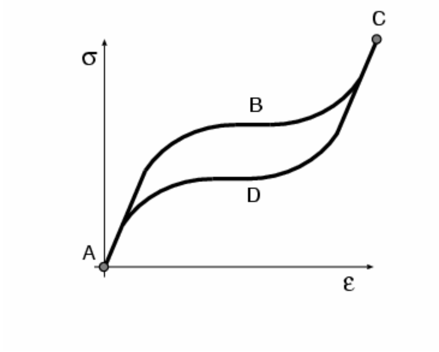
Liang and Rogers. *Journal of Engineering Mathematics* 1992

Sato and Tanaka. *Res. Mech.* 1988

Boyd and Lagoudas. *Int. J. Plasticity* 1995

Liang and Rogers. *Journal of Engineering Mathematics* 1992

Superelastic 1D SMA model

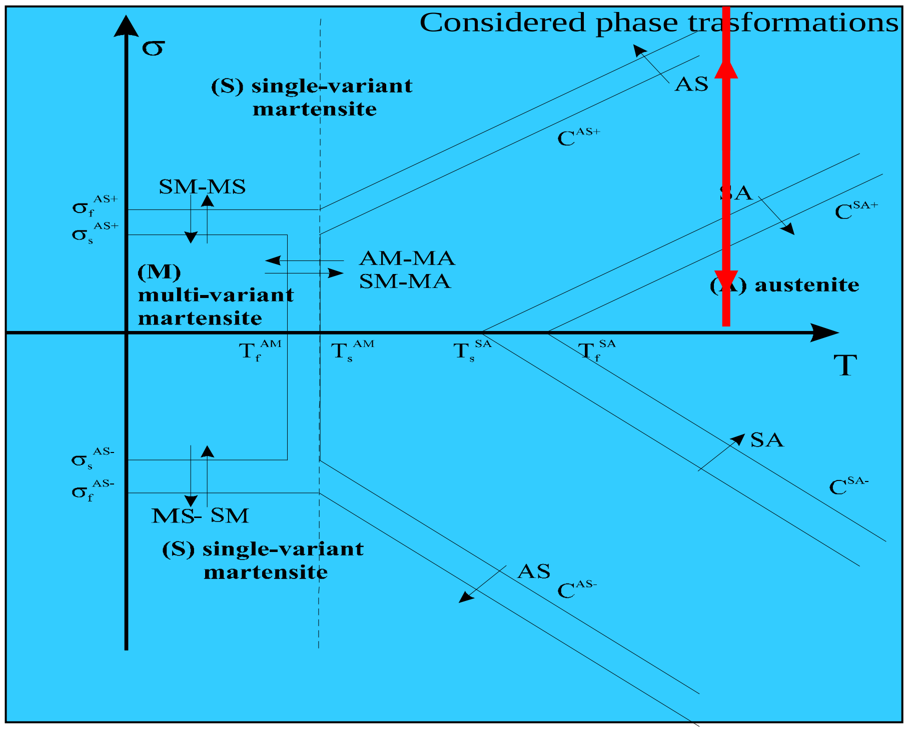


ξ_A austenite volume fraction

ξ_S single variant martensite volume fraction

$$\xi_A + \xi_S = 1$$

$$\xi_S = 1 - \xi_A$$



Stress-strain relationship

$$\sigma = E(\varepsilon - \xi_S \varepsilon_L^{(\pm)})$$

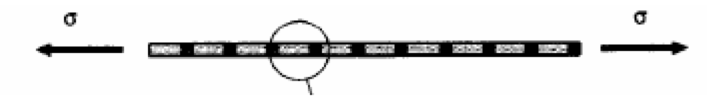
$\varepsilon_L^{(\pm)}$ maximum residual strain

Evolution of the Young modulus

$$E(\xi) = \frac{E^A E^S}{E^S + \xi(E^A - E^S)}$$

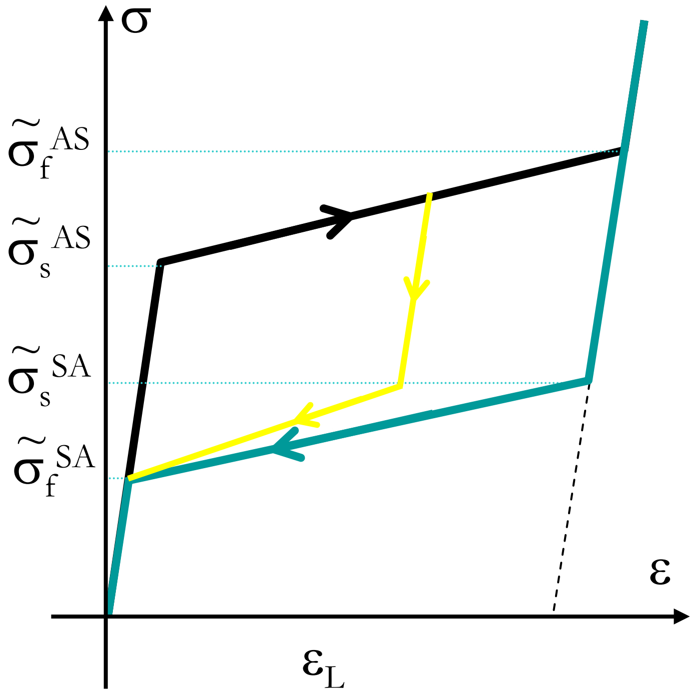
E^A austenite Young modulus

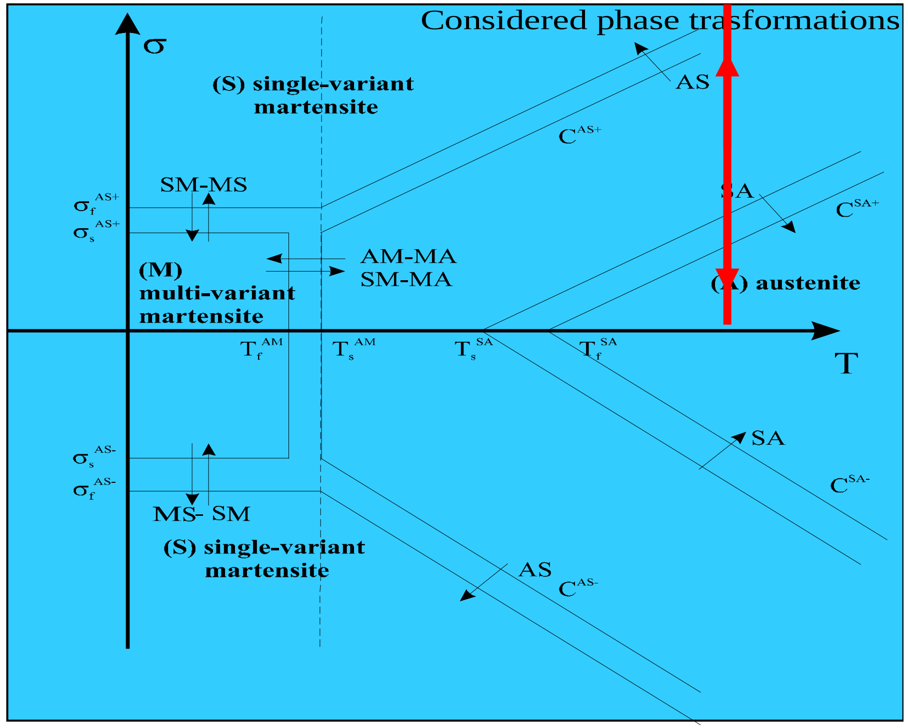
E^S martensite Young modulus



Reuss homogenization

Evolution of the internal parameter





austenite - martensite evolution

when $\sigma_s^{AS(\pm)} \le |\sigma| \le \sigma_f^{AS(\pm)}$

$$\dot{\xi}_S = -(1-\xi_S)\frac{1}{|\sigma| - \sigma_f^{AS(\pm)}}\dot{\sigma}$$

martensite - austenite evolution

when $\sigma_s^{SA(\pm)} \le |\sigma| \le \sigma_f^{SA(\pm)}$

$$\dot{\xi}_S = \xi_S \frac{1}{|\sigma| - \sigma_f^{SA(\pm)}}\dot{\sigma}$$

Note

when $\sigma_s^{AS(\pm)} \le \sigma \le \sigma_f^{AS(\pm)}$

$$\dot{\xi}_S = -(1-\xi_S)\frac{1}{\sigma - \sigma_f^{AS(\pm)}}\dot{\sigma} \qquad \frac{\dot{\xi}_S}{(1-\xi_S)} = -\frac{1}{\sigma - \sigma_f^{AS(\pm)}}\dot{\sigma}$$

$$\int_0^{\xi_{S,t}} \frac{\dot{\xi}_S}{(1-\xi_S)} = -\int_{\sigma_s^{AS(\pm)}}^{\sigma_t} \frac{1}{\sigma - \sigma_f^{AS(\pm)}}\dot{\sigma}$$

$$-\ln\left(1-\xi_{S,t}\right) = -\ln\left(\sigma_f^{AS(\pm)} - \sigma_t\right) + \ln\left(\sigma_f^{AS(\pm)} - \sigma_s^{AS(\pm)}\right)$$

$$\ln\left(\frac{1}{1-\xi_{S,t}}\right) = \ln\left(\frac{\sigma_f^{AS(\pm)} - \sigma_s^{AS(\pm)}}{\sigma_f^{AS(\pm)} - \sigma_t}\right) \qquad \frac{1}{1-\xi_{S,t}} = \frac{\sigma_f^{AS(\pm)} - \sigma_s^{AS(\pm)}}{\sigma_f^{AS(\pm)} - \sigma_t}$$

$$\xi_{S,t} = \frac{\sigma_t - \sigma_s^{AS(\pm)}}{\sigma_f^{AS(\pm)} - \sigma_s^{AS(\pm)}}$$

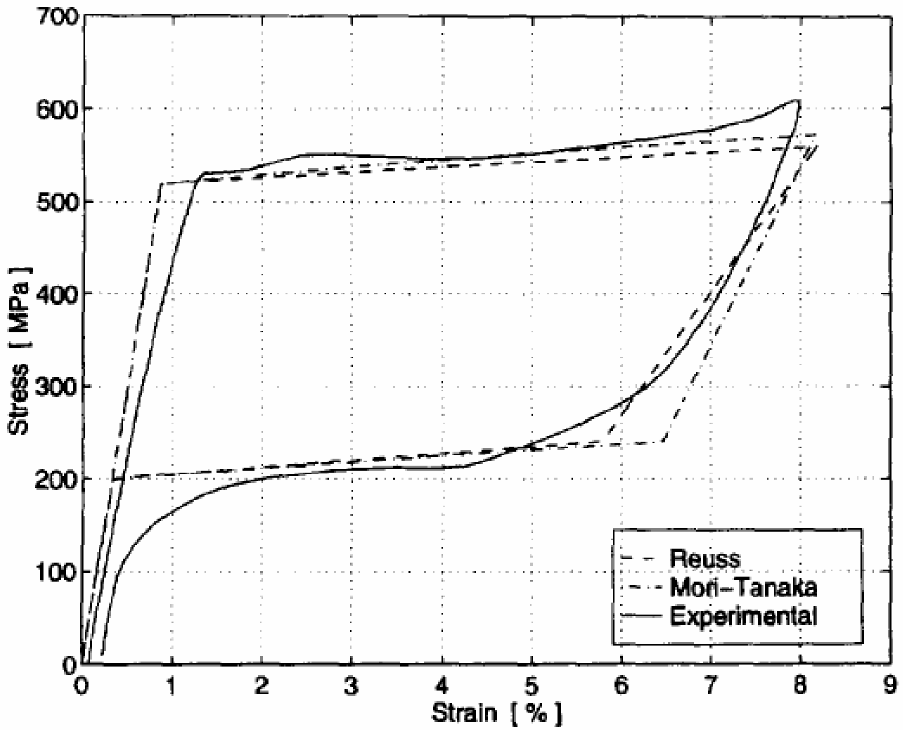
Results

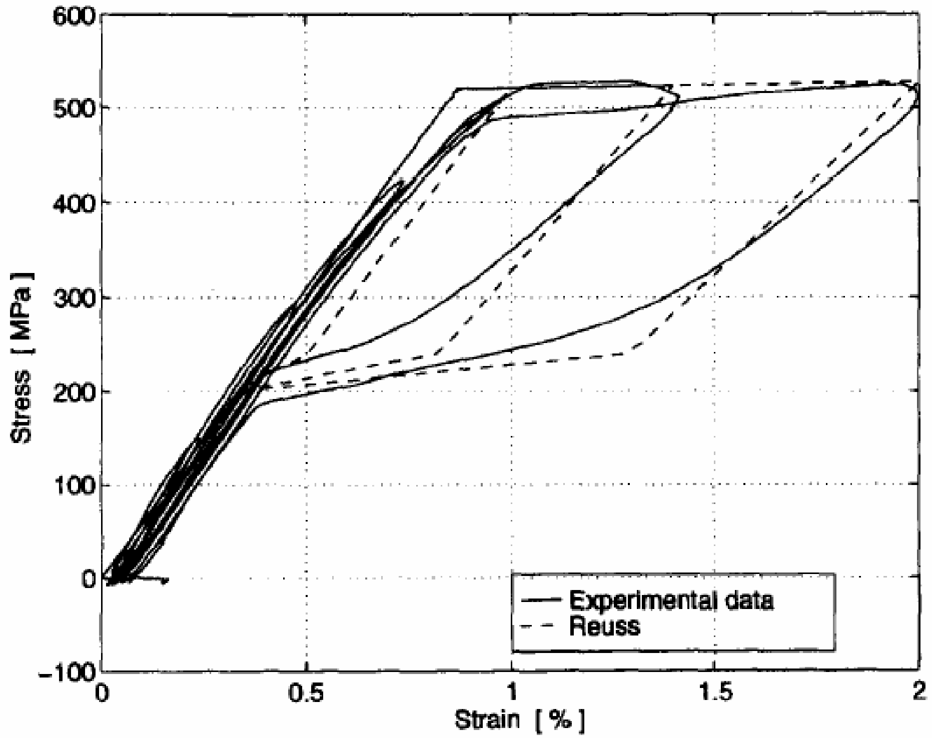
NDC Ni-Ti alloy provided by Nitinol Device & Components

$$E_A = 60{,}000 \text{ MPa}, \quad E_S = 7500 \text{ MPa}, \quad \varepsilon_L = 8\%$$

$$\sigma_s^{AS} = 520 \text{ MPa}, \quad \sigma_f^{AS} = 600 \text{ MPa}$$

$$\sigma_s^{SA} = 240 \text{ MPa}, \quad \sigma_f^{SA} = 200 \text{ MPa}$$



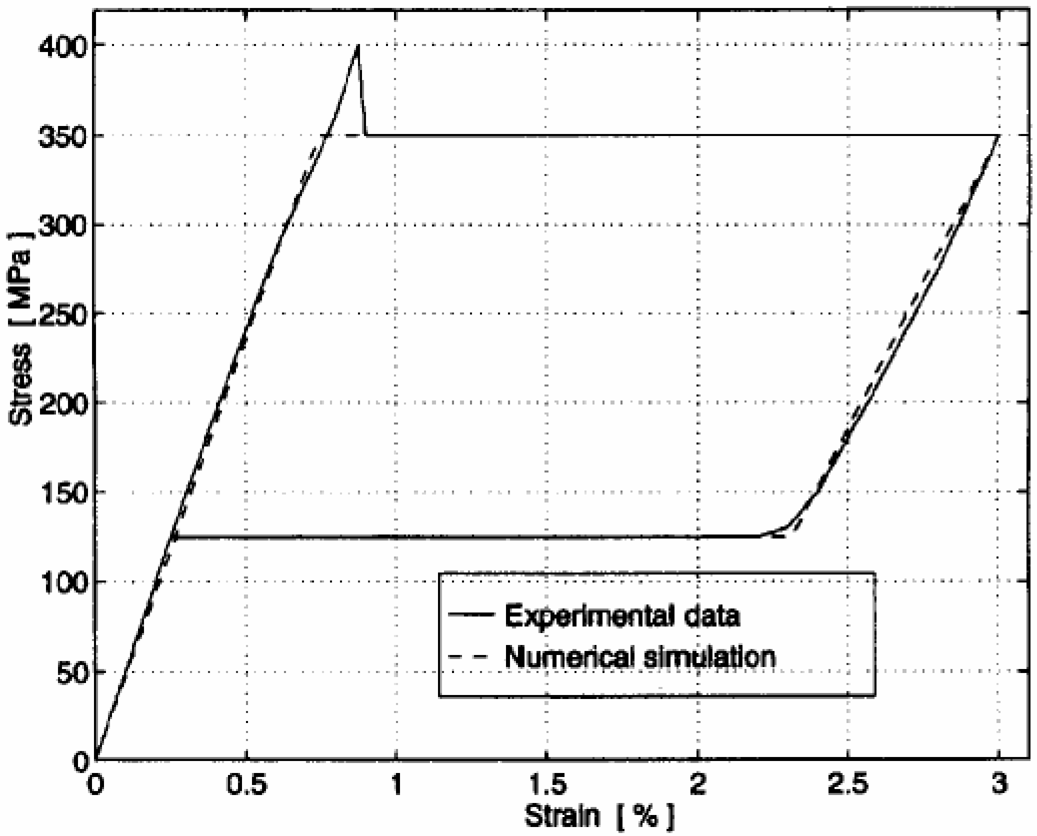


GAC Ni-Ti alloy provided by GAC International Inc.

$$E_A = 47{,}000\ \text{MPa}, \quad E_S = 17{,}000\ \text{MPa}, \quad \varepsilon_L = 8\%$$

$$\sigma_s^{AS} = 350\ \text{MPa}, \quad \sigma_f^{AS} = 350\ \text{MPa}$$

$$\sigma_s^{SA} = 125\ \text{MPa}, \quad \sigma_f^{SA} = 125\ \text{MPa}$$

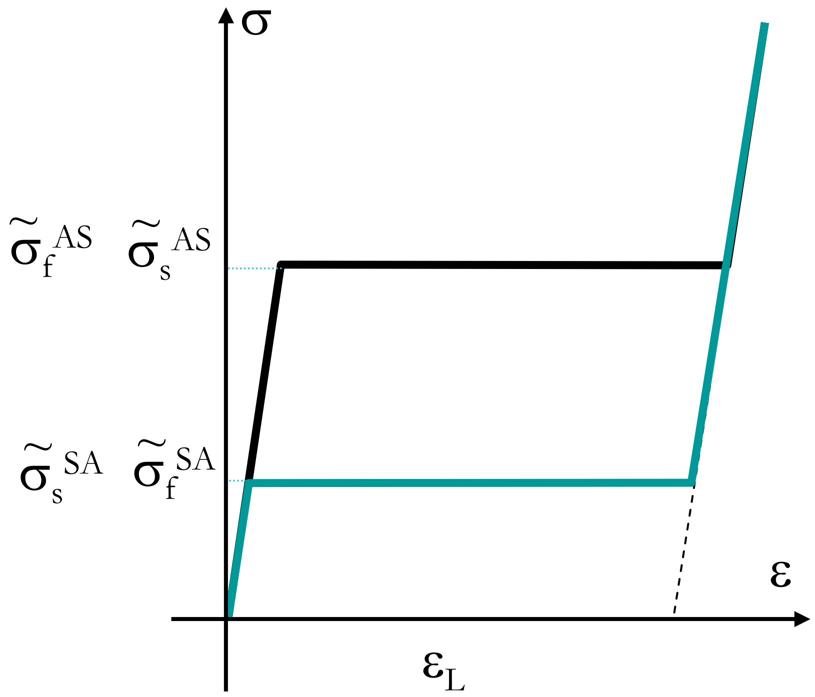


Beam problem

1. *Kinematics*
2. *Constitutive model*
3. *Equilibrium equations*
4. Numerical procedures (fiber model)

Appendix: An analytical solution

- **equal material response in tension and compression,**
- **equal elastic properties between austenite and martensite,**
- **equal initial and final values for the phase transformations,**
- **pure bending loading state.**



$$y_1: \quad E\varepsilon(y_1) = \sigma^{AS} \quad \text{with} \quad 0 \le y_1 \le \frac{h}{2}$$

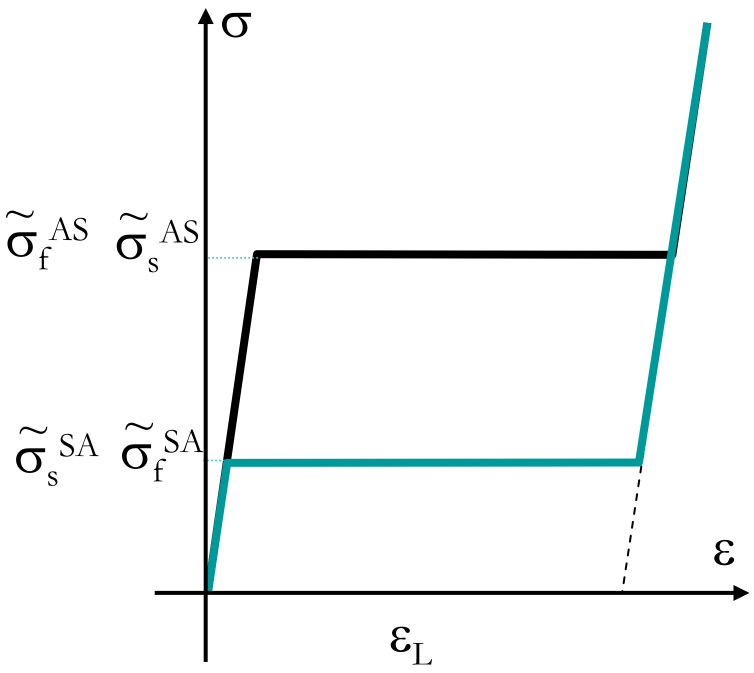
$$y_2: \quad E\left[\varepsilon(y_2) - \varepsilon_L\right] = \sigma^{AS} \quad \text{with} \quad 0 \le y_2 \le \frac{h}{2}$$

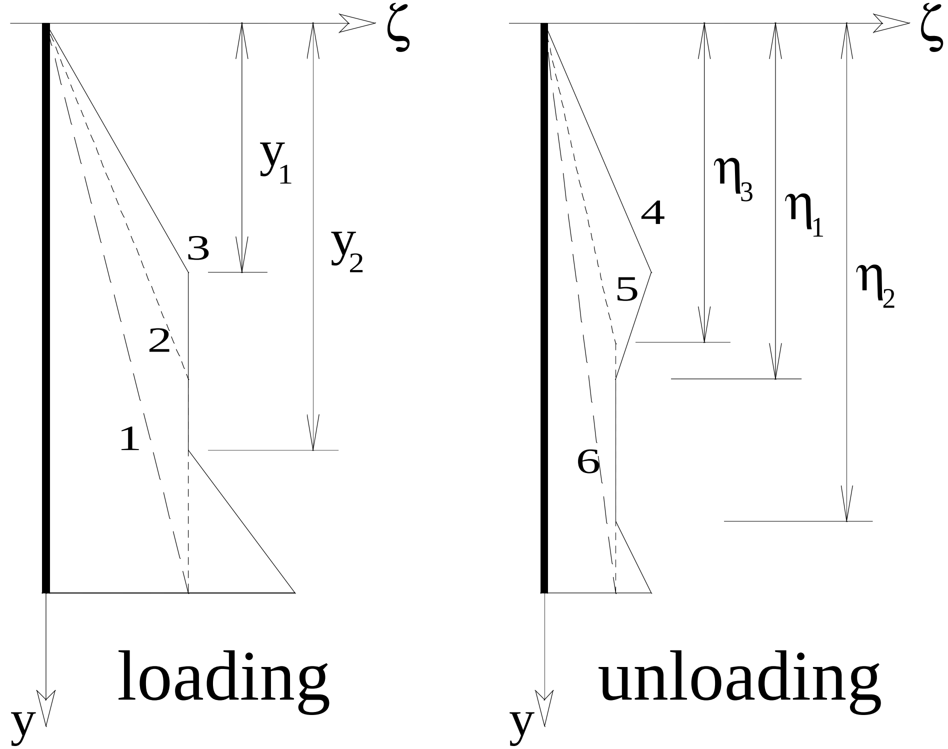
$$\eta_1: \quad E\left[\varepsilon(\eta_1) - \varepsilon_p(\eta_1)\right] = \sigma^{SA} - \sigma^{AS} \quad \text{with} \quad y_{1,p} \le \eta_1 \le y_{2,p}$$

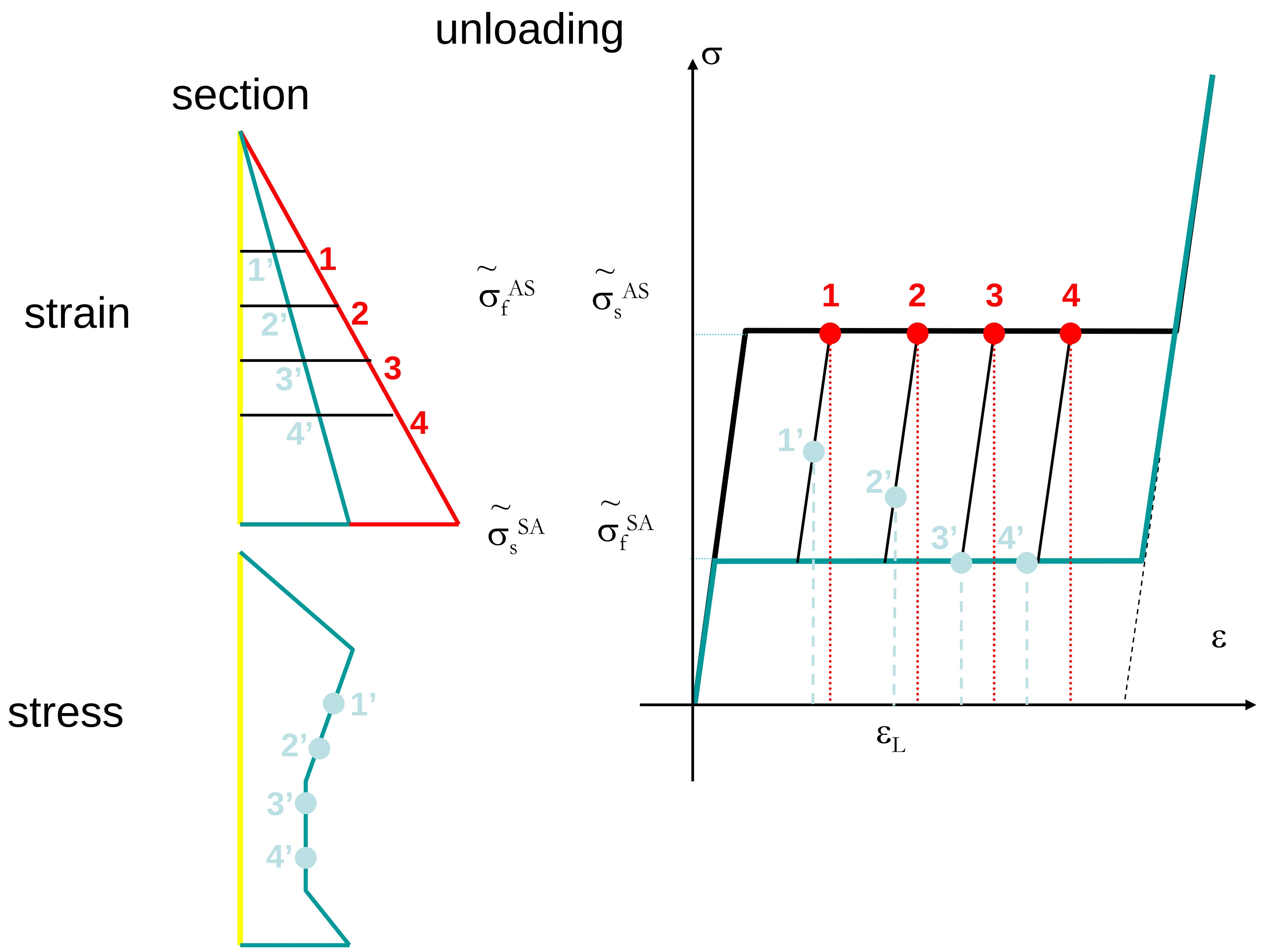
$$\eta_2: \quad E\left[\varepsilon(\eta_2) - \varepsilon_L\right] = \sigma^{SA} \quad \text{with} \quad y_{2,p} \le \eta_2 \le \frac{h}{2}$$

$$\eta_3: \quad E\varepsilon(\eta_3) = \sigma^{SA} \quad \text{with} \quad y_{1,p} \le \eta_3 \le \frac{h}{2}$$

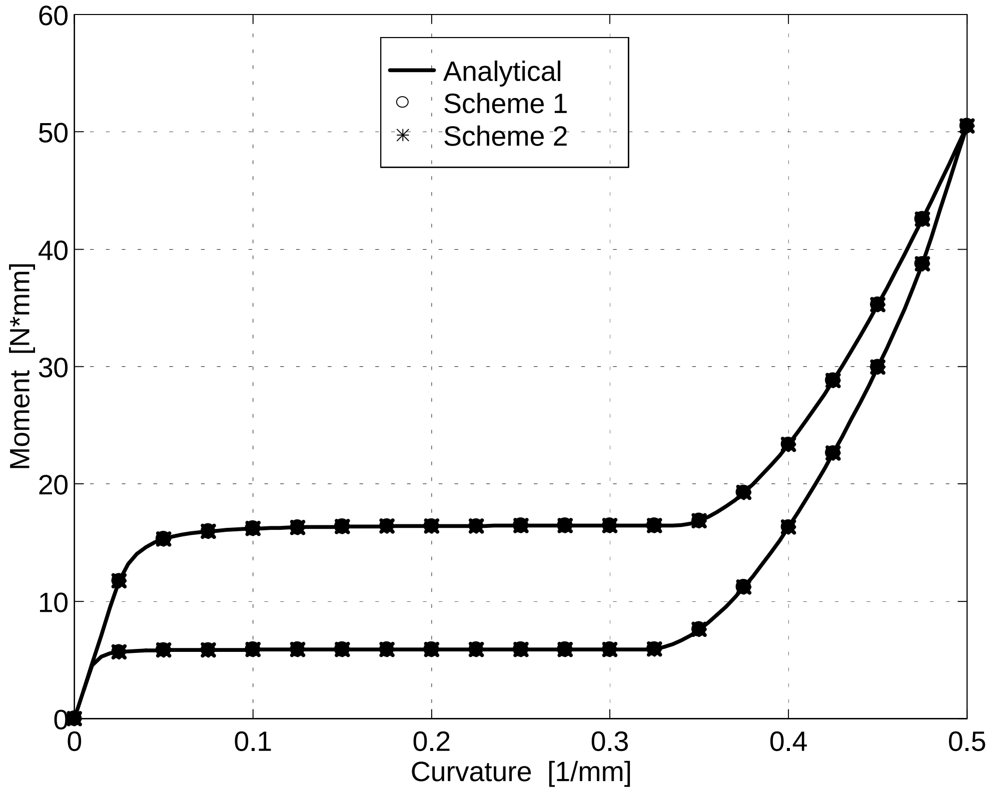
rectangular cross-section *b*×*h*






unloading
section
strain
stress
1
2
3
4
1'
2'
3'
4'
$\tilde{\sigma}_f^{AS}$
$\tilde{\sigma}_s^{AS}$
$\tilde{\sigma}_s^{SA}$
$\tilde{\sigma}_f^{SA}$
σ
ε
ε_L

Comparison between analytical and numerical solutions (*GAC Ni-Ti alloy*)



bending moment versus the curvature

Stress distribution in the cross-section evolution during the loading process

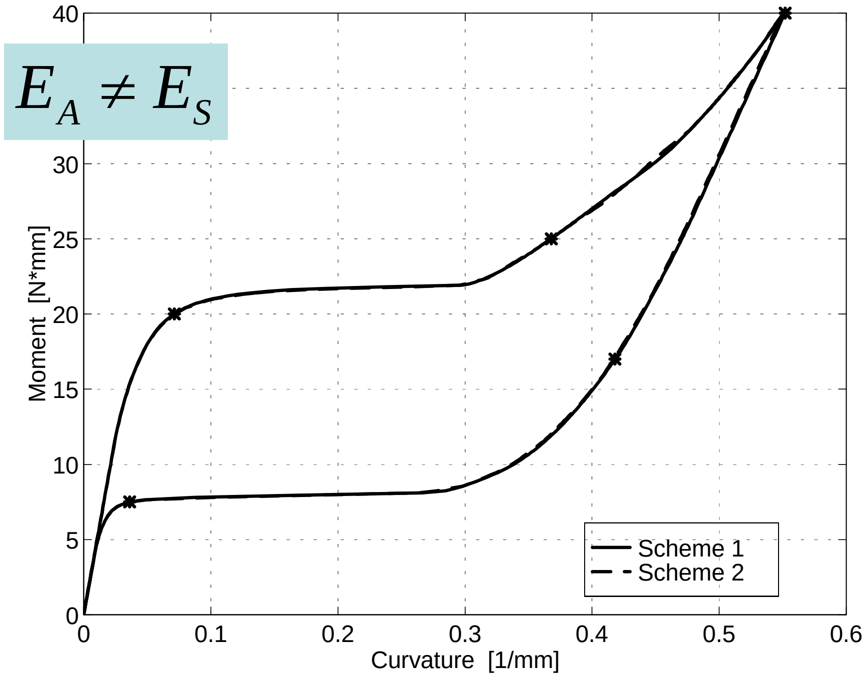
1. no phase transformation, linear elastic material response, neutral axis at zero.

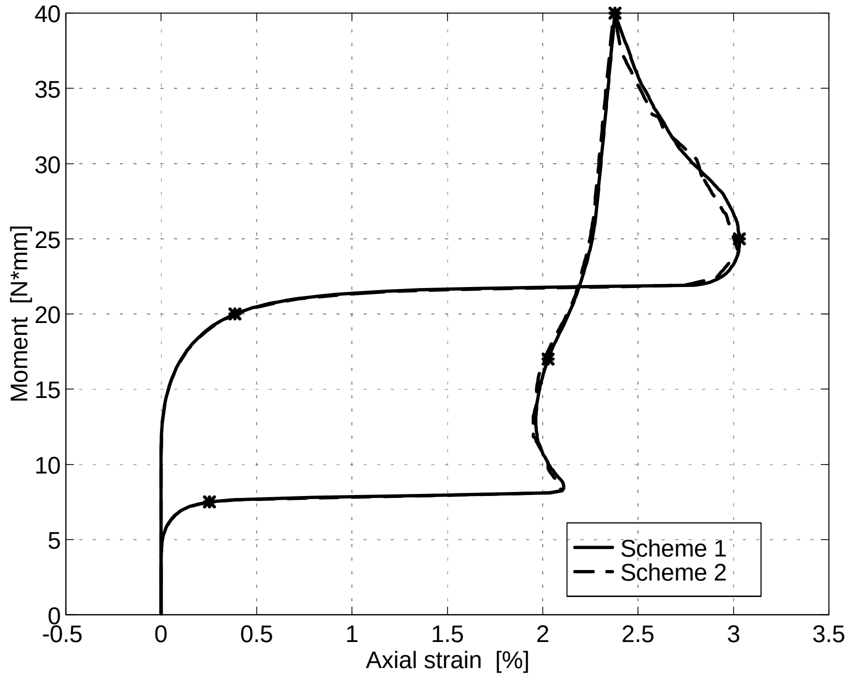
2. phase transformation on the part of the cross-section in traction (stress transformation higher in compression than in tension), then transformation in compression; neutral axis starts to move downward and axial deformation shows up.

3. material restiffening when the phase transformation is completed in traction; progressive upward movement of the neutral axis, reduction of the axial deformation..

Unloading process stress pattern more and more complex: combination of the neutral axis movement and the different response in tension and in compression.

curvature and axial strain versus the bending moment

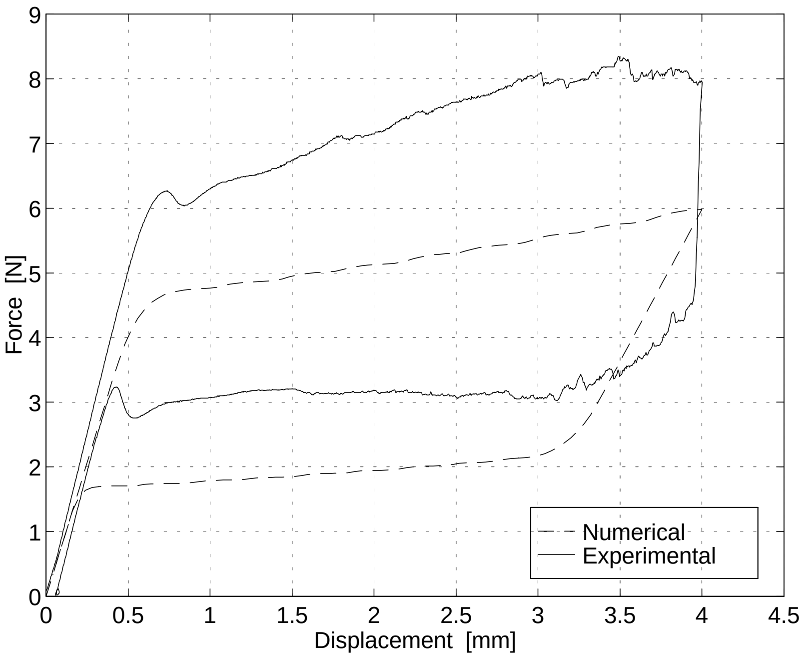




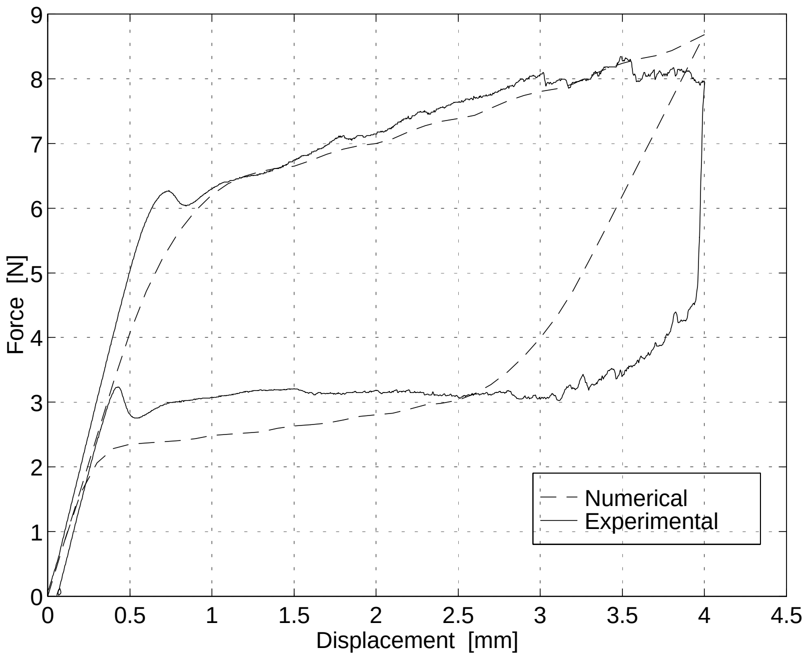
Three point bending

GAC wire

equal response in tension and compression



different material response in tension and in compression

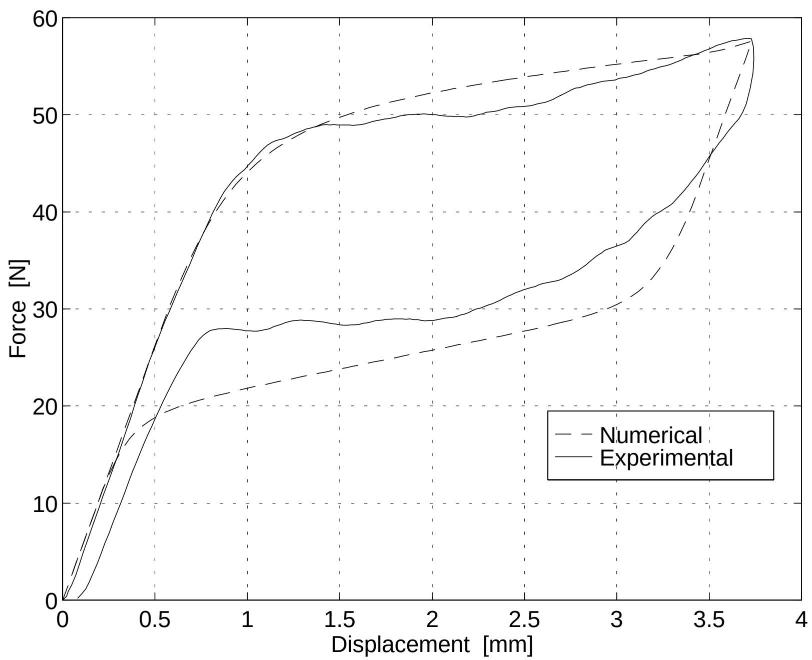


Four point bending

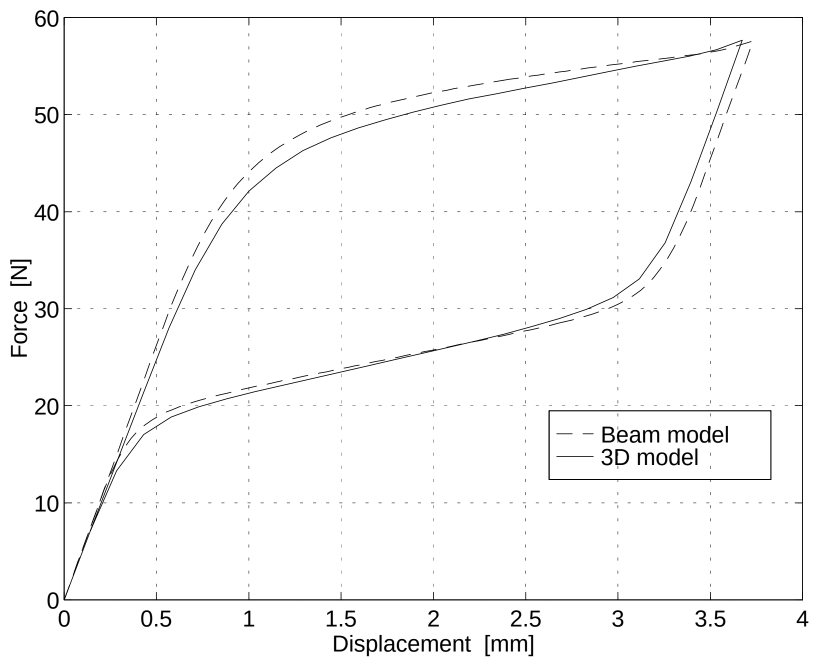
NDC circular wire

equal properties in traction and in compression and equal elastic moduli between austenite and martensite.

comparison between numerical and experimental results



comparison between beam and three-dimensional analyses



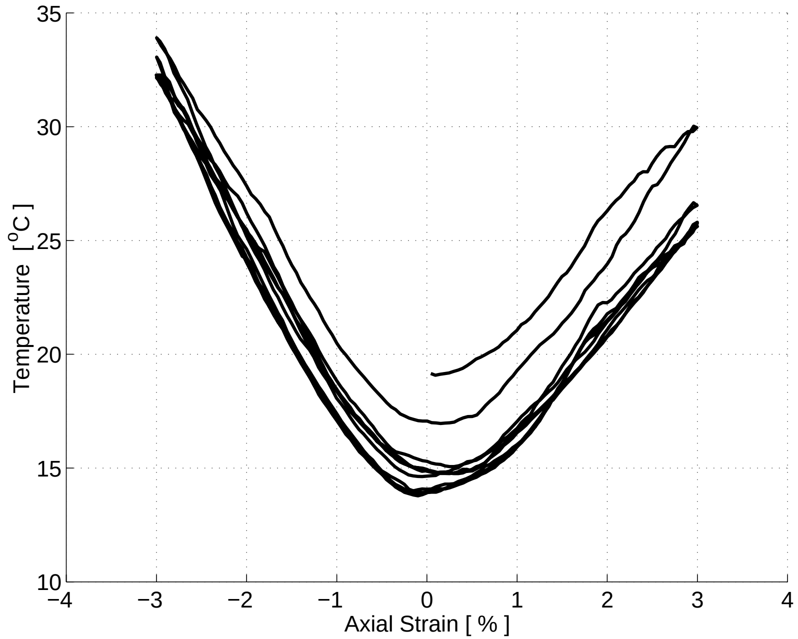
applied force F versus midspan inflection

Thermomechanical modeling of SE-SMA wires

Experimental data reveal:

- material response characterized by an hysteresis, indicating a mechanical dissipation;
- superelastic response associated to significative temperature variations.
- austenite-martensite phase transformations occur in well defined stress-temperature ranges and they can be induced through mechanical loading-unloading patters, through thermal cooling-heating patterns or through combined mechanical-thermal patterns;

SMAs show strong thermo-mechanical constitutive coupling.



typical superelastic response in tension and compression [Lim and McDowell (1999)]

Single-phase material free-energy [Raniecki – Bruhns (1991)]

$$\psi(\varepsilon^{el},T)=\psi_0(T)+\psi^{el}(\varepsilon^{el})-(T-T_0)s^{el}(\varepsilon^{el},T)+C\left[(T-T_0)-T\log\frac{T}{T_0}\right]$$

$\psi_0 = u_0 - Ts_0$ free-energy in the natural or reference state $(T=T_0, \varepsilon^{el}=0)$

u_0 s_0 internal energy and entropy in the reference state.

$\psi^{el}=\frac{1}{2}E\left(\varepsilon^{el}\right)^2$ free-energy increase due to the elastic strain for $T=T_0$

$$s=-\frac{\partial\psi}{\partial T}=s_0+s^{el}(\varepsilon^{el})+C\log\frac{T}{T_0}$$ entropy

$s^{el}(\varepsilon^{el})=\varepsilon^{el}E\alpha$ entropy increase due to the elastic strain for $T=T_0$

α thermal expansion factor $C=T\frac{\partial s}{\partial T}$ heat capacity

final form of the free-energy

$$\psi=u_0-Ts_0+\frac{1}{2}E\left(\varepsilon^{el}\right)^2-(T-T_0)\varepsilon^{el}E\alpha+C\left[(T-T_0)-T\log\frac{T}{T_0}\right]$$

Material undergoing phase transformation

$$\psi_0 = \left[u_A - Ts_A\right] - \xi_S\left[\Delta u - T\Delta s\right] + \xi_S\left(1-\xi_S\right)\left[u_{int} - Ts_{int}\right]$$

u_A s_A internal energy and the entropy of the austenite

Δu Δs internal energy difference and the entropy difference between the austenite and the martensite

u_S internal energy and the entropy of the martensite

$u_{int} - Ts_{int}$ interaction term

free-energy form for a material undergoing a solid-solid PT

$$\psi = \frac{1}{2}E\left(\varepsilon - \varepsilon_L\xi_S\, s\right)^2 - \left(T - T_0\right)\left(\varepsilon - \varepsilon_L\xi_S\, s\right)E\alpha + C\left[\left(T - T_0\right) - T\log\frac{T}{T_0}\right]$$

$$+\left[\left(u_A - Ts_A\right) - \xi_S\left(\Delta u - T\Delta s\right)\right]$$

stress and entropy
$$\sigma = \frac{\partial \psi}{\partial \varepsilon^{el}} = \frac{\partial \psi}{\partial \varepsilon} = E(\varepsilon - \varepsilon_L \xi_S \; s) - E\alpha(T - T_0)$$

$$s = -\frac{\partial \psi}{\partial T} = s_A - \xi_S \Delta s + C \log \frac{T}{T_0} + (\varepsilon - \varepsilon_L \xi_S \; s) E\alpha$$

heat equation
$$C\dot{T} + \operatorname{div} q = b$$

$$b = H_{\text{tmc}} + D_{\text{mec}}$$
heat source

heat production associated to the thermomechanical coupling

$$H_{\text{tmc}} = T \frac{\partial^2 \psi}{\partial T \partial \varepsilon} \dot{\varepsilon} + T \frac{\partial^2 \psi}{\partial T \partial \xi_S} \dot{\xi}_S = T\left[-E\alpha\dot{\varepsilon} + \left(\varepsilon_L E\alpha + \Delta s\right)\dot{\xi}_S \right]$$

heat production associated to dissipative mechanical processes

$$D_{\text{mec}} = \sigma\dot{\varepsilon} - \left(\frac{\partial \psi}{\partial \varepsilon} \dot{\varepsilon} + \frac{\partial \psi}{\partial \xi_S} \dot{\xi}_S \right) = \left(\varepsilon_L \,|\, \sigma \,| - T\Delta s + \Delta u \right) \dot{\xi}_S$$