2.2 STRAIN GAGES

2.2.1 Fundamentals: Piezoresistive Effect

Strain gages are based on the variation of resistance of a conductor or semiconductor when subjected to a mechanical stress.

For a wire having length l, cross section A, and resistivity ρ, the electric resistance is

$$R = \rho \frac{l}{A} \tag{2.5}$$

When it is stressed longitudinally, each of the three quantities that affect R change and therefore R undergoes a change given by

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{dA}{A} \tag{2.6}$$

The change in length that results when a force F is applied to a wire, within the elastic limit (Figure 2.4), is given by Hooke's law,

$$\sigma = \frac{F}{A} = E\varepsilon = E\frac{dl}{l} \tag{2.7}$$

where E is a constant for each material, called *Young's modulus*, σ is the mechanical stress, and ε is the strain (unit deformation). ε has no dimensions, but to improve clarity, it is usually given in "microstrains" (1 microstrain = 1 $\mu\varepsilon$ = 10^{-6} m/m).

Consider a wire that in addition to a length l has a transverse dimension t. As a result of longitudinal stress not only does l change but t does too. The relationship between both changes is given by Poisson's law

$$\nu = -\frac{dt/t}{dl/l} \tag{2.8}$$

where ν is the Poisson ratio. It usually ranges from 0 to 0.5; it is, for example, 0.17 for cast iron, 0.303 for steel, and 0.33 for aluminum and copper. Note that for the volume to remain constant, it should be $\nu = 0.5$.

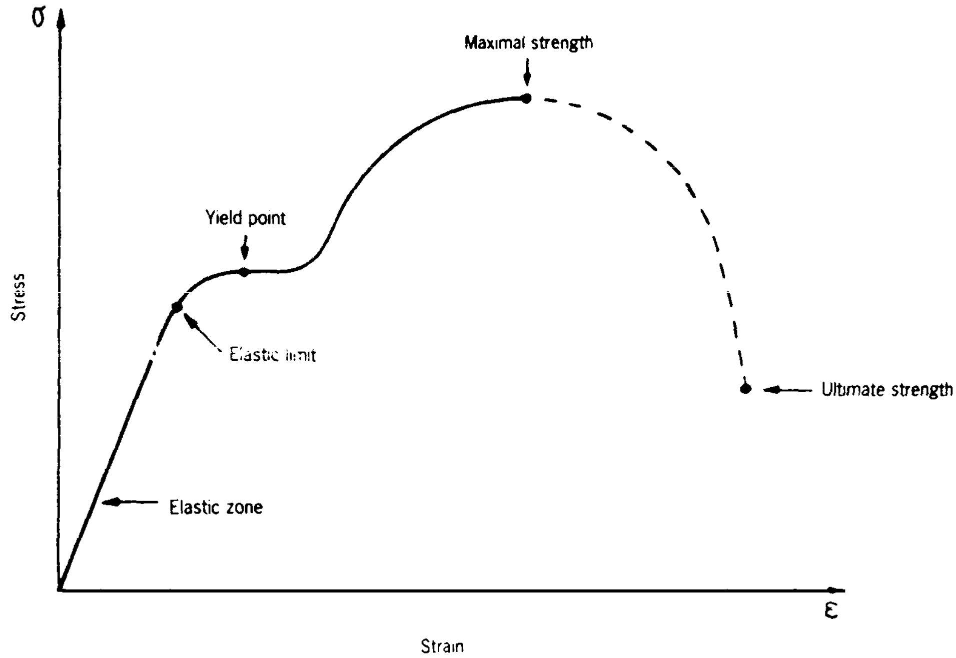

FIGURE 2.4 Stress-strain curve for metal. The elastic region has been greatly enlarged.

For a wire of circular cross section of diameter D, we have

$$A = \frac{\pi D^2}{4} \tag{2.9}$$

$$\frac{dA}{A} = \frac{2dD}{D} = -\frac{2\nu dl}{l} \tag{2.10}$$

The change in resistivity as a result of a mechanical stress is called the *piezoresistive effect*. In metals the percent changes of resistivity and volume are proportional:

$$\frac{d\rho}{\rho} = C\frac{dV}{V} \tag{2.11}$$

where C is the Bridgman constant, which ranges from 1.13 to 1.15 for the usual alloys from which strain gages are made, and is 4.4 for platinum. By using (2.10), the change in volume can be expressed as

$$V = \frac{\pi l D^2}{4} \tag{2.12}$$

$$\frac{dV}{V} = \frac{dl}{l} + \frac{2dD}{D} = \frac{dl}{l}(1 - 2\nu) \tag{2.13}$$

Therefore if the material is isotropic, within the elastic limit, (2.6) leads to

$$\frac{dR}{R} = \frac{dl}{l}[1 + 2\nu + C(1 - 2\nu)] = G\frac{dl}{l} \tag{2.14}$$

where G is the gage factor, defined as the factor inside the square brackets. From the given values for the different parameters, $G \approx 2$, with the exception of platinum for which $G \approx 6$.

Therefore for small variations the resistance of the metallic wire is

$$R = R_0(1 + x) \tag{2.15}$$

where R_0 is the resistance when there is no applied stress and $x = G\varepsilon$. The change in resistance does not exceed 2%.

When a semiconductor is stressed, the piezoresistive effect dominates. *The relationship between resistance and strain is* [2]:

For a given p-type material

$$\frac{dR}{R_0} = 119.5\varepsilon + 4\varepsilon^2 \tag{2.16}$$

For a given n-type material

$$\frac{dR}{R_0} = -110\varepsilon + 10\varepsilon^2 \tag{2.17}$$

where R_0 is the resistance at 25°C for a constant supply current and no applied stress.

Thus there is a relationship between the change in the electric resistance of a material and the strain it experiences. If the relationship between that strain and the force causing it is known [3], from the measurement of resistive changes it is possible to infer the applied forces and the quantities that produce those forces in a sensor. A resistor arranged to sense a strain constitutes a strain gage. This method has proved to be very useful for many years. There are many limitations we must consider concerning this measurement principle to obtain valid information.

First, the applied stress should not exceed the elastic limit of the gage. It does not exceed 4% of gage length and ranges, approximately, from 3000 $\mu\varepsilon$ for semiconductor gages to 40,000 $\mu\varepsilon$ for metal gages.

Second, the measurement will be correct only if all the stress is transmitted to the gage. This is achieved by carefully bonding the gage with an elastic adhesive that must be also stable with time and temperature. At the same time the gage must be electrically insulated from the object it adheres to and be protected from the environment.

We assume that all strains are in the same plane, that is, that there is no stress in any direction perpendicular to the gage wires. To have a significant electric resistance for the gage, it consists of a grid containing several longitudinal segments connected by smaller transverse segments having a larger cross section (Figure 2.5). Thus the transverse sensitivity is only from 1% to 2% of the longitudinal sensitivity. Figure 2.6 shows the conventional method for installing a strain gage.

Temperature is a source of interference for several reasons. It affects the resistivity of the material, its dimensions, and the dimensions of the support material. Thus once the gage is placed on a surface, any change in temperature will cause a change in resistance even before applying any mechanical force. In metal strain gages this change can be as large as 50 $\mu\varepsilon/°C$.

This interference may be compensated by using the method of opposing inputs. We can use dummy gages. These are gages equal to the sensing gages and placed near them in order to experience the same temperature change but without experiencing any mechanical stress. Section 3.4.4 will explain how they are placed in the measuring circuit. To avoid excessive differential strains, strain gages are available for each material to be tested, that is, gages having a thermal expansion coefficient similar to that of the material.

Temperature effects are very pronounced in semiconductor strain gages. In self-compensated gages, the increase in resistivity with increasing temperature is compensated by a decrease in resistance due to the expansion of

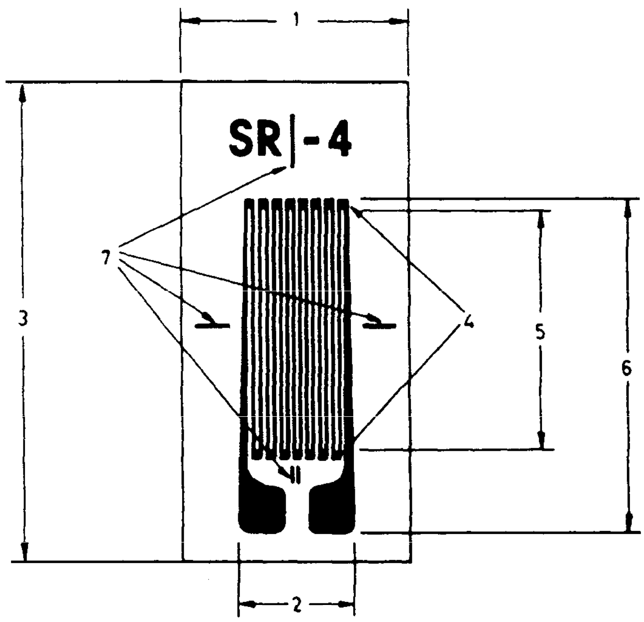

FIGURE 2.5 Parameters for a foil strain gage (from BLH Electronics): 1 matrix width; 2 grid width; 3 matrix length; 4 tabs; 5 active grid length; 6 overall gage length; 7 alignment marks.

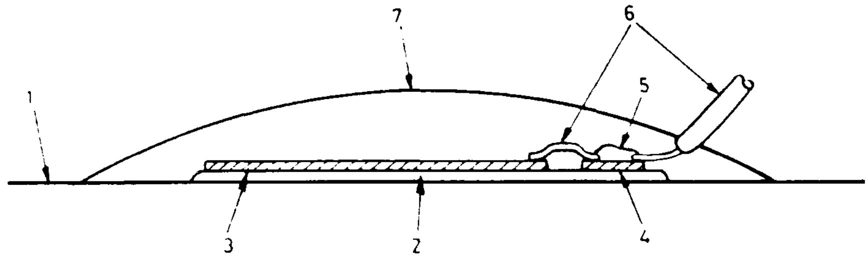

FIGURE 2.6 Installation of a foil strain gage (from BLH Electronics): 1 substrate material; 2 adhesive; 3 strain gage; 4 solder terminals; 5 solder; 6 lead wires; 7 environmental barrier.

the backing material. With this method it is possible to obtain thermal deformations of only 5 $\mu\varepsilon$/°C.

In order to measure strain gage resistance, we must pass an electric current through it, and the resulting power dissipation causes heating. The maximal current is 25 mA for metal gages if the base material is a good heat conductor (steel, copper, aluminum) and 5 mA if it is a poor heat conductor (plastic, wood). The permissible power increases with the gage area and ranges from 770 mW/cm^2 to 150 mW/cm^2, depending on the backing. In semiconductor strain gages the maximal power dissipation is 250 mW.

Another source of interference is the thermoelectric force appearing at the junction between dissimilar metals (Section 6.1.1). If the gage is supplied by a dc voltage, these may produce a net voltage which adds to the voltage due to strain. Thermoelectric forces can be detected by reversing the supply polarity: If they are present, the output voltage will change. Thermoelectric forces can be avoided by applying the intrinsic insensitivity method, selecting appropriate materials, filtering, or supplying the gages with ac voltage.

Strain gages should ideally be very small in order to measure strain at a given point. In practice, the dimensions are finite, and we assume that the measured "point" is at the gage geometric center. When vibrations are measured, their wavelength must be longer than the gage. If, for example, a gage has a useful length of 5 mm and the measurement is done in steel where sound velocity is approximately 5900 m/s, then the frequency for one wavelength to equal one gage length is 5900 m/s/0.005 m $\approx$ 1 MHz. To keep 10% of the wavelength equal to the gage length, we calculate the maximal measurable frequency is about 100 kHz (1 MHz/10).

When measuring a nonuniform surface, such as concrete, we should measure an average strain in order to avoid any inaccuracies due to discontinuity in the surface. In this case a large gage should be used.

Silicon strain gages have been shown to be also light dependent, although optical effects are probably negligible under conventional illumination conditions [12].

In spite of all these possible limitations, strain gages are some of the most popular sensors because of their small size, high linearity, and low impedance.

2.2.2 Types and Applications

Strain gages are made of different metals such as the alloys constantan, advance, karma, and nichrome and also of semiconductors such as silicon and germanium. Strain gages can be either bonded or unbonded. If bonded, the strain gage is chosen to have the same thermal expansion coefficient as the backing. For tactile sensors in robots, conductive elastomers may form the strain gage. Figure 2.7 shows different types of strain gages. Bonded metal strain gages can be made with paper backing from parallel wire or from foil. Figure 2.7 shows some of the many different available paper-backed configurations. There are models for diaphragms, for torsion, to determine minimal and maximal stress and its direction (multiple rosettes), as well as measure other internal forces.

Table 2.2 gives some of the usual characteristics of metal and semiconductor strain gages. The gage factor is determined by sampling because strain gages cannot be reused. The manufacturer gives the probable value for G and its tolerance.

Strain gages can be applied to the measurement of any quantity that by the use of the appropriate sensor we convert into a force capable of producing deformations of 10 μm and even lower.

Figure 2.8 shows several applications of strain gages for measuring force and torque. The arrangement of strain gages in measurement bridges will be analyzed in Section 3.4. By using similar strain gage methods, it is possible to measure pressure, flow, acceleration, and so on.

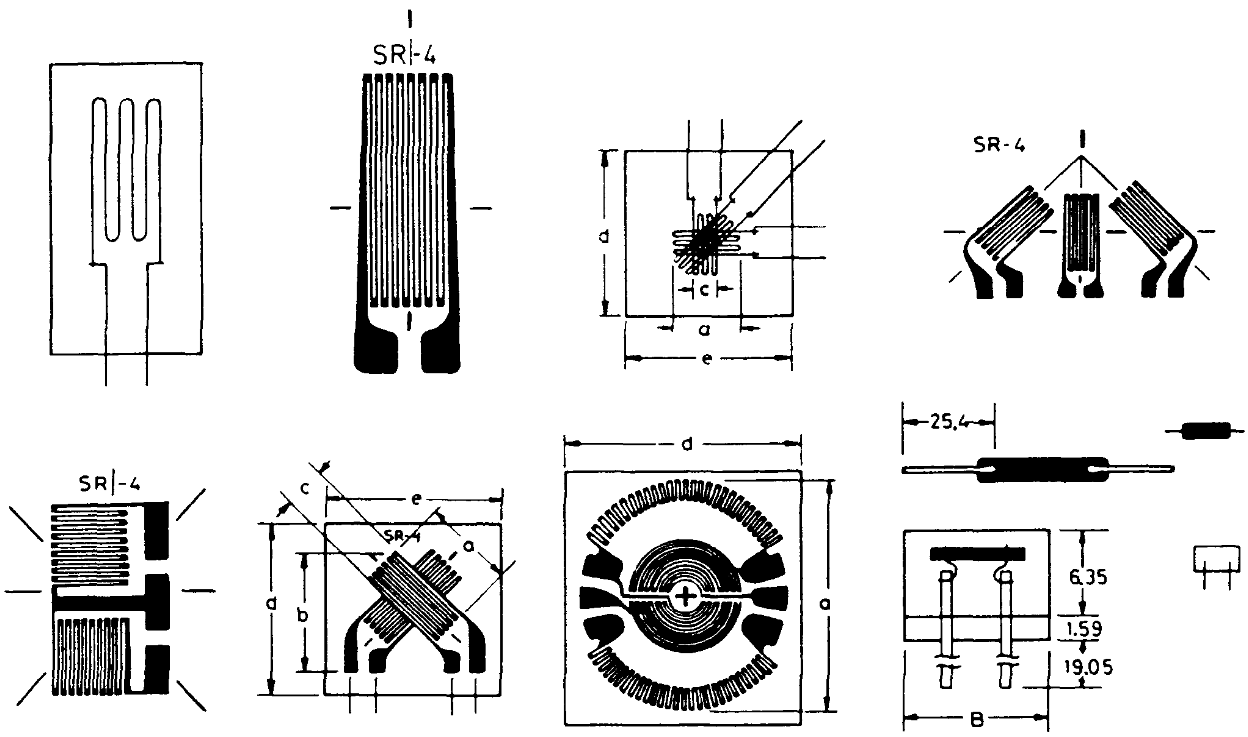

FIGURE 2.7 Several bonded and unbonded metal and semiconductor strain gages (from BLH Electronics).

TABLE 2.2 Typical Characteristics of Metal and Semiconductor Strain Gages

Parameter	Metal	Semiconductor
Measurement range	0.1 to 40,000 $\mu\varepsilon$	0.001 to 3000 $\mu\varepsilon$
Gage factor	1.8 to 2.35	50 to 200
Resistance, Ω	120, 350, 600, . . . , 5000	1000 to 5000
Resistance tolerance	0.1% to 0.2%	1% to 2%
Size, mm	0.4 to 150 Standard: 3 to 6	1 to 5

A unique application of the piezoresistive effect is the measurement of very high pressures (1.4 GPa to 40 GPa) by means of manganin gages. Manganin is an alloy (84% Cu, 12% Mn, 4% Ni) which has a very low temperature coefficient. If a manganin wire is subjected to a pressure from all directions, it exhibits a coefficient of resistance of 280 $\mu\Omega/\Omega$/kPa, its change of resistance thus giving information about the applied pressure.

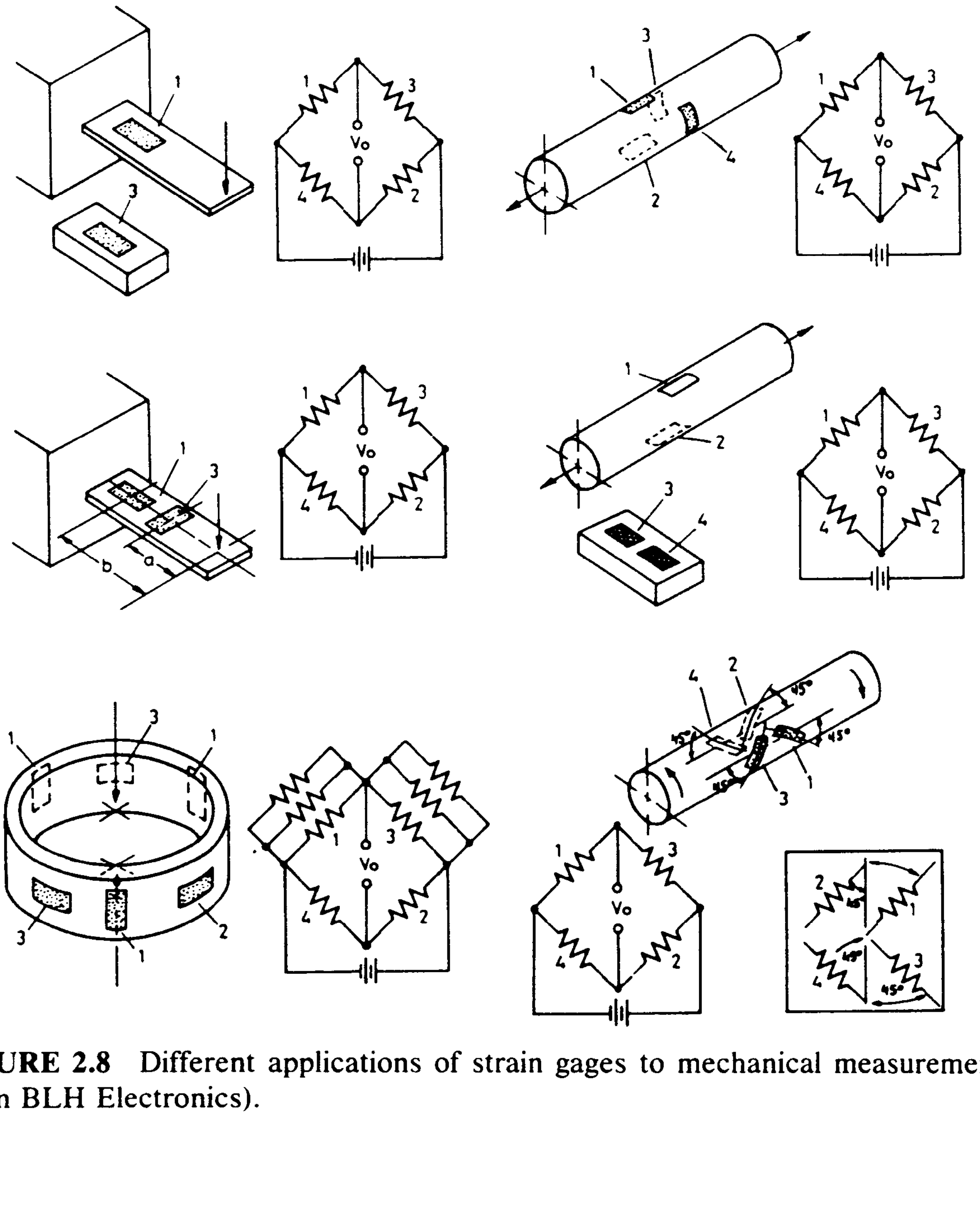

FIGURE 2.8 Different applications of strain gages to mechanical measurements (from BLH Electronics).

3.3 WHEATSTONE BRIDGE: BALANCE MEASUREMENTS

The Wheatstone bridge measurement method, shown in Figure 3.15, is based on a feedback system, either electric or manual, whose function is to adjust the value of a standard resistor until the current through the galva-

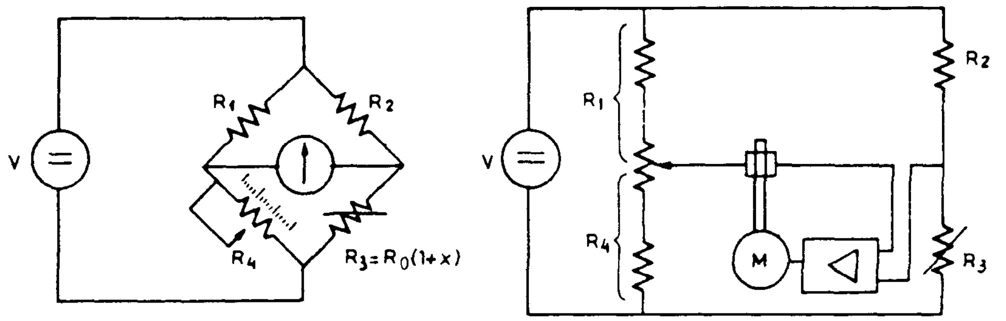

FIGURE 3.15 Comparison measurement method for a Wheatstone bridge. The adjustment to reach balance can be performed manually or automatically.

nometer or other null indicator is zero. Once the balance condition has been achieved, we have

$$R_3 = R_4 \frac{R_2}{R_1} \tag{3.28}$$

That is, changes in R_3 are directly proportional to the corresponding changes we have to produce in R_4 in order for the bridge to be balanced. This measurement method can also be used as a polarity detector because the output is positive or negative depending on whether x is greater or less than a given threshold.

The condition (3.28) is reached independent of the power supply voltage or current and its possible variations. It does not depend on the type of detector or its impedance. Even more, it does not need to be linear because it must only indicate the balance condition. From (3.28) we can also deduce that the supply and the detector can interchange their positions without affecting the measurement. Figure 3.16 shows an arrangement for eliminating the influence that the contact resistance in the adjustable arm has on the measurement.

If the sensor is placed far from the bridge, we must consider the presence of long lead wires whose resistance adds to the sensor resistance. Their

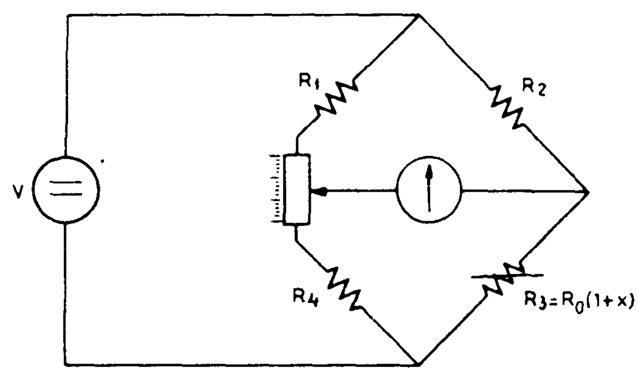

FIGURE 3.16 Wheatstone bridge arrangement to cancel the effect of contact resistance on the balance.

values can be very high if low temperature coefficient resistive conductors such as constantan and manganin are used. Conversely, if copper wires are used because of its higher conductivity, then temperature changes can result in important errors.

This problem can be solved by using the Siemens or three-wire method shown in Figure 3.17*a*. Wires 1 and 3 must be equal and be subjected to the same thermal changes. The characteristics of wire 2 are not important as in the balance condition no current flows through the bridge central arm. The relative error in the measurement of R_3 is

$$\varepsilon = \frac{R_3 - R_4R_2/R_1}{R_3} = \frac{R_w}{R_3}\left(\frac{R_4}{R_1} - 1\right) \tag{3.29}$$

Figure 3.17*b* shows an alternative circuit with the same objective. The error in this case is given by an expression similar to (3.29). In both cases the

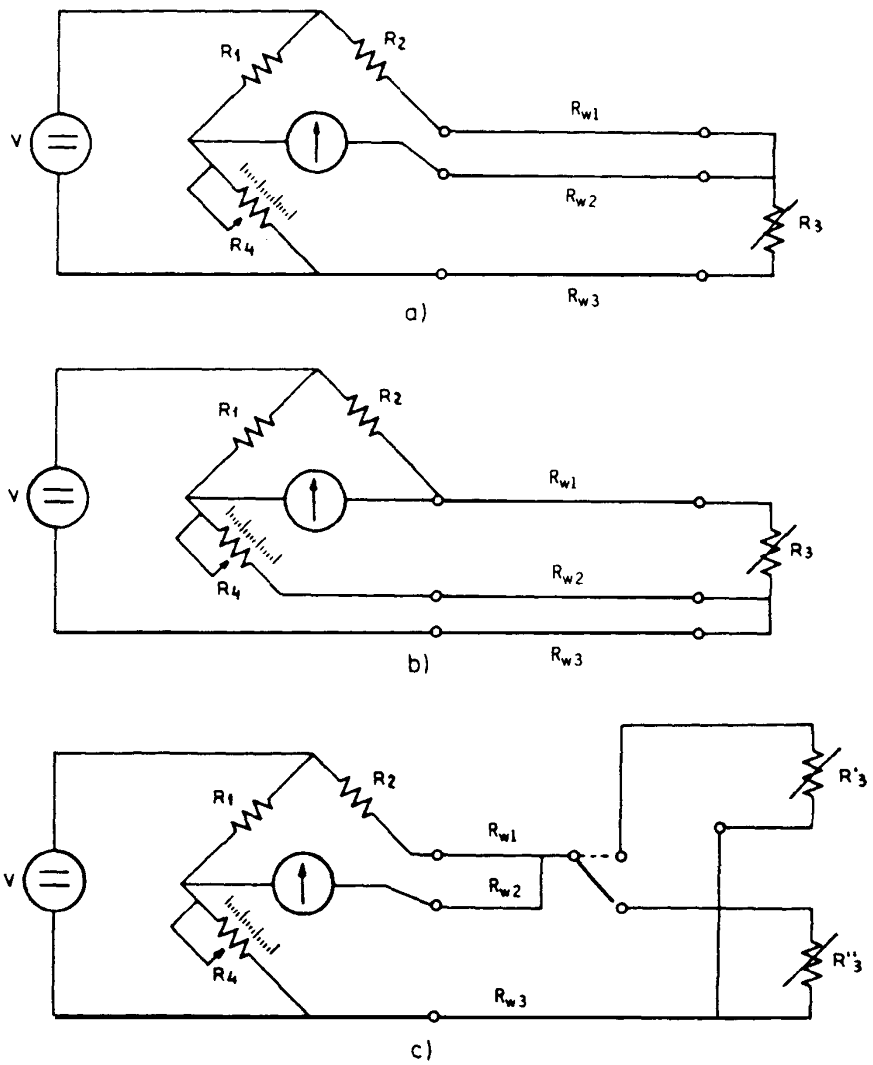

FIGURE 3.17 Siemens or three-wire method for measuring with a Wheatstone bridge when long leads are used.

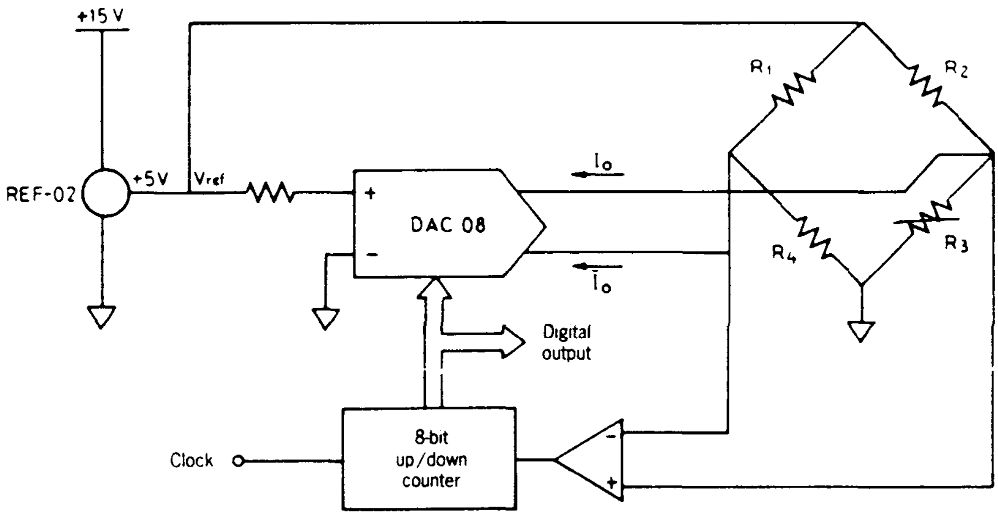

FIGURE 3.18 Wheatstone bridge using the comparison method with automatic balance and digital output.

error decreases when R_3 is large with respect to R_w. Figure 3.17*c* shows how to apply this method to several sensors without using more than a single set of three long wires.

The application of the null method to dynamic measurements depends on the availability of a fast enough automatic balancing system. Figure 3.18 shows such a method [3]. It is based on a digital-to-analog converter whose analog outputs are in the form of two complementary current sources. That is, in addition to a current corresponding to the digital input, it outputs another current corresponding to the complementary digital input. This way the sum of both currents is always a constant regardless of the digital input.

In Figure 3.18 any imbalance of the bridge output exceeding the comparator threshold modifies the converter outputs, via the up-down counter, so that one of them sinks the additional current necessary to keep the drop in voltage constant in both voltage dividers. At the same time the other converter output reduces the amount of current it sinks, thus contributing to the voltage balance which is reached independently of the sign of the change experienced by the sensor. The output of the system is then the digital word present at the input of the converter in order for the bridge to remain balanced.

3.4 WHEATSTONE BRIDGE: DEFLECTION MEASUREMENTS

3.4.1 Sensitivity and Linearity

A common way of obtaining an electric signal in a Wheatstone-bridge-based measurement is with the deflection method. Instead of measuring the action needed to restore balance in the bridge, this method measures the voltage difference between both bridge outputs or the current through a detector placed in the central arm. Using the notation of Figure 3.19, if the bridge is balanced when $x = 0$, which is the usual situation, we define a parameter k,

$$k = \frac{R_1}{R_4} = \frac{R_2}{R_0} \tag{3.30}$$

We measure the voltage difference between both sides to obtain

$$V_o = V\left(\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4}\right) = V\frac{kx}{(k+1)(k+1+x)} \tag{3.31}$$

Thus the output voltage is proportional to the changes in resistance only when $x \ll k + 1$. If we interpret the output voltage as being proportional to changes in R_3, the real sensitivity is

$$S = \frac{V_o}{xR_0} = \frac{Vk}{R_0}\frac{1}{(k+1)(k+1+x)} \tag{3.32}$$

The maximal value for this sensitivity as a function of k is obtained by taking $dS/dk = 0$, which yields

$$k^2 = 1 + x \tag{3.33}$$

By taking the second derivative, we can verify that this point is a maximum. If the measured output is the current through the central arm or if instead of supplying the bridge with a constant voltage, a constant current is supplied, then the corresponding conditions for maximal sensitivity are different. They are given in Table 3.1 [4].

Therefore, for this case the condition for maximal sensitivity is obtained for a value of k that may be not high enough to achieve the desired linearity, as indicated by (3.31). In particular, the maximal sensitivity when x takes very small values will be obtained when $k = 1$. Figure 3.20 shows the sensi-

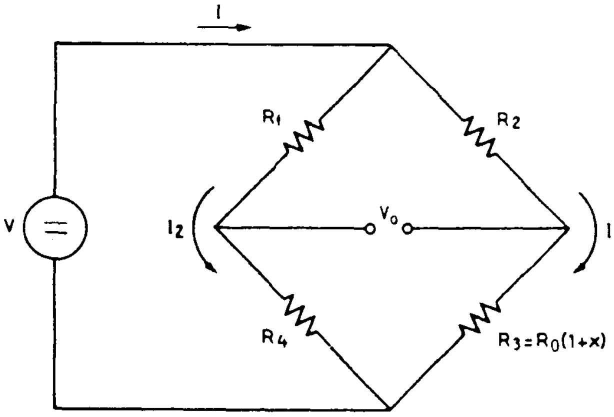

FIGURE 3.19 Wheatstone bridge using the deflection method.

TABLE 3.1 Optimal Dimensions of Bridge Resistances

Primary Condition	Secondary Condition	Maximal Occurrence for	Approximate Maximum
Maximal current sensitivity dI_m/dR_3	I constant	$m = \infty$	$R_2 \gg R_3$
		$n^2 = q + 1$	
	V constant	$m^2 = q/(1+q)$	$R_4 \ll R_3$
		$n = 0$	
	I_1 constant	$m = \infty$	
		$n = 0$	$R_4 \ll R_3 \ll R_2$
Maximal voltage sensitivity dV_m/dR_3	I constant	$m = \infty$	$R_4 \gg R_3$
		$n = \infty$	$R_2 \gg R_3$
	V constant	$m = 1$	$R_3 = R_2$
	I_1 constant	$m = \infty$	$R_2 \gg R_3$

Note: $m = R_2/R_3$, $n = R_4/R_3$, $p = R_1/R_3$, $q = R_m/R_3$; R_m is the resistance of the measuring instrument placed in the central arm of the bridge.

Source: K. S. Lion, *Elements of Electrical and Electronic Instrumentation*, © 1975. Reprinted by permission of McGraw-Hill, New York.

tivity as a function of k for the case $x = 0.001$ in a constant voltage supplied bridge when its output voltage is measured (equation 3.32).

If the bridge is supplied by a constant current I, the output voltage is then

$$V_m = I\frac{kx}{2(k+1)+x} \tag{3.34}$$

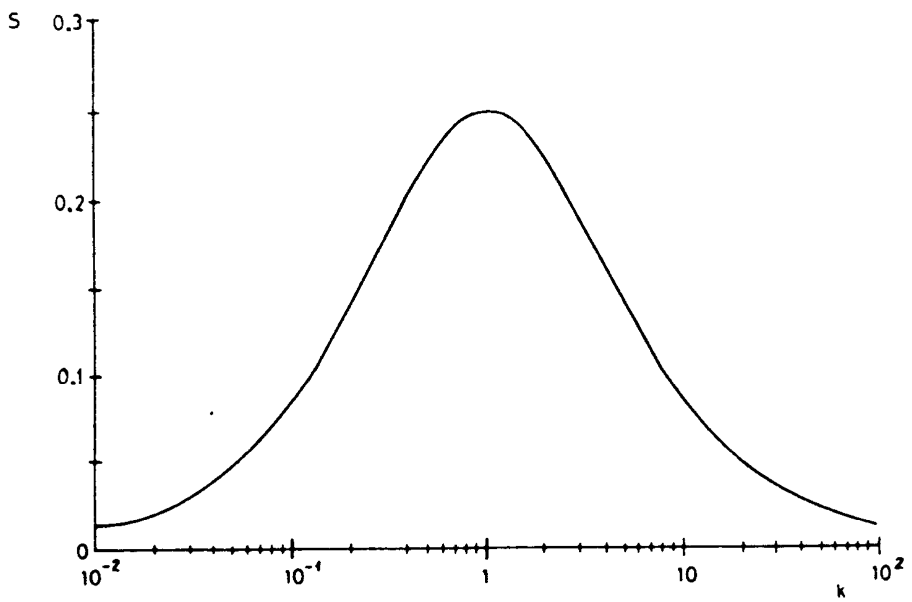

FIGURE 3.20 Sensitivity in a resistance bridge supplied by a constant current as a function of the parameter k and $x = 0.001$, when the output voltage is measured (equation 3.32).

To have an approximately linear output, it is necessary that $x \ll 2(k + 1)$ and that $x \ll 4$ when $k = 1$. But a constant current source is difficult to build without using electronic circuitry, and the resulting improvement does not always warrant it. Linearity is not necessary to achieve good accuracy. What matters is the repeatability of the results. But the interpretation of the output is always easier when it is proportional to the measured quantity. Thus we are interested in the linearity predicted by equations (3.31) and (3.34).

For the case of metal strain gages, x seldom exceeds 0.01. Therefore, unless a very high linearity is desired, the presence of x in the denominator of (3.31) can be ignored. But for resistance thermometers x can be much higher. For a Pt100-based thermometer, for example, at 125°C the resistance has changed from the 100-Ω value at 25°C to 140 Ω. For these cases we have the following alternatives:

1. Restrict the measurement range to a narrow zone where the maximal nonlinearity is compatible with the required measurement accuracy.
2. Work with a reduced sensitivity by making $k = 10$, or even higher, and compensating part of the sensitivity loss by means of an increase in supply voltage. This option is limited by the maximal power dissipation of the sensors. By supplying short duty cycle rectangular voltages, it is possible to have high peak output values with low power supply voltages.
3. Linearize the bridge output voltage by using analog or digital techniques.

   Figure 3.21 shows the effect of k on the linearity of the bridge in Figure 3.19 when the output is in voltage form and the range for x is $-1 < x < +1$.

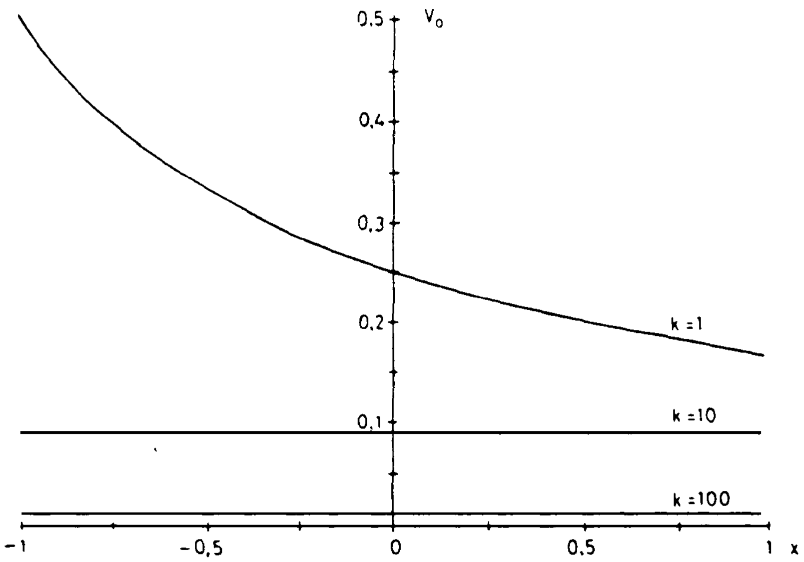

FIGURE 3.21 Effect of the value for k in a resistance bridge such as the one in Figure 3.19, for $-1 < x < +1$.

Example: A temperature ranging from $-10°C$ to $+50°C$ is to be measured giving a corresponding output voltage from -1 to $+5$ V, with an error less than 0.5% of the reading plus 0.2% of full-scale output. The sensor available is an RTD having a resistance of 100 Ω at 0°C, temperature coefficient 0.4%/°C at 0°C, and thermal dissipation constant under the measurement circumstances of 5 mW/K. The proposed solution is a voltage-supplied dc bridge whose output voltage is connected to an ideal amplifier. Calculate the value for bridge resistors and the supply voltage needed. Calculate the theoretical sensitivity for the bridge and the gain for the amplifier.

For the circuit in Figure 3.19, if we take $R_3 = R_T = R_0(1 + \alpha T)$, where T is the incremental temperature above the temperature at which R_0 was measured, equation (3.31) takes the form

$$V_o = V \frac{k\alpha T}{(k + 1 + \alpha T)(k + 1)}$$

If we assume this voltage is linear, we desire an ideal response of

$$V_I = V \frac{k\alpha T}{(k + 1)^2}$$

The relative error due to nonlinearity will be

$$\varepsilon = \frac{V_o - V_I}{V_o} = \frac{-\alpha T}{k + 1}$$

Therefore the error depends on the measured temperature and increases when the temperature does. At 0°C we want a 0-V output. We choose $R_0 =$ 100 Ω. Then we can choose α to be the specified value for the thermal coefficient (4×10^{-3}). The maximal relative error will be at $T = 50°C$. We want $\varepsilon < 5 \times 10^{-3}$, therefore

$$\frac{4 \times 10^{-3} \times 50}{k + 1} < 5 \times 10^{-3}$$

This requires $k > 39$. Then the values for the other bridge resistors are, $R_4 = 100\ \Omega$, $R_1 = R_2 = 3900\ \Omega$. If larger values were chosen for R_1 and R_2, then the sensitivity would decrease.

The sensitivity is also determined by the supply voltage for the bridge. This is limited by sensor self-heating. Recognizing that self-heating implies a nearly constant error yields

$$P = \left(\frac{V}{R_2 + R_T}\right)^2 R_T < 2 \times 10^{-3} \times 50°C \times 5\ \text{mW/°C} = 0.5\ \text{mW}$$

We determine the temperature at which the maximal power will be dissipated by deriving the expression for the dissipated power, equating it to zero, and determining if the second derivative is negative at that temperature. The result is that maximal heating occurs when $R_T = R_2$. In the measurement range this condition will never occur. R_T is always lower than R_2. The worst case in our measurement range will be $T = 50°C$ because at this temperature R_T reaches its maximal value of 120 Ω.

These conditions yield

$$V < \left(\frac{5 \times 10^{-4}}{120}\right)^{1/2} 4020 = 8.2 \text{ V}$$

If $V = 8$ V is chosen for convenience, then the sensitivity will be 0.78 mV/°C. The gain to obtain a 5 V output when $T = 50°C$ will be

$$G = \frac{5}{0.78 \times 10^{-3} \times 50} = 128.2$$

3.4.2 Analog Linearization of Resistive Sensor Bridges

To obtain a voltage proportional to any size changes in one of the resistances in a Wheatstone bridge, we can either modify the structure of the bridge or perform analog processing of the output voltage for the unmodified bridge.

In the first case Figure 3.22 shows that a constant current passes through the sensor. We subtract the resulting voltage drop from that across a fixed resistance R_0. For an ideal op amp the output is

$$V_o = -V\frac{x}{2} \tag{3.35}$$

The supply voltage is limited by power dissipation of the bridge resistors. However, the most important limitation is the requirement of having five

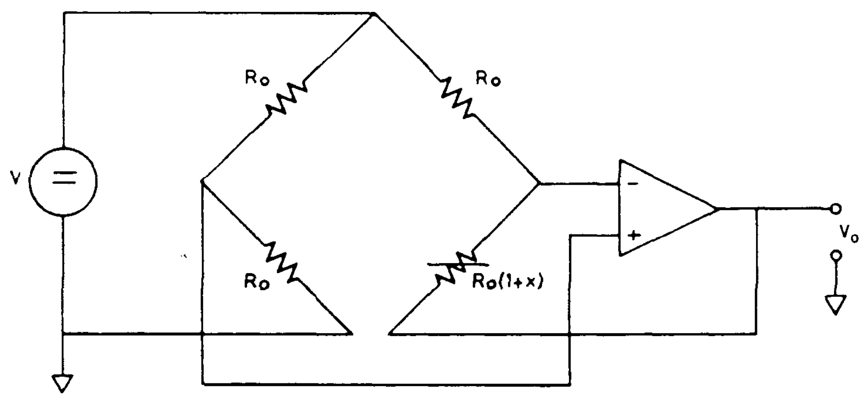

FIGURE 3.22 Analog linearization of a resistance bridge with five accessible terminals.

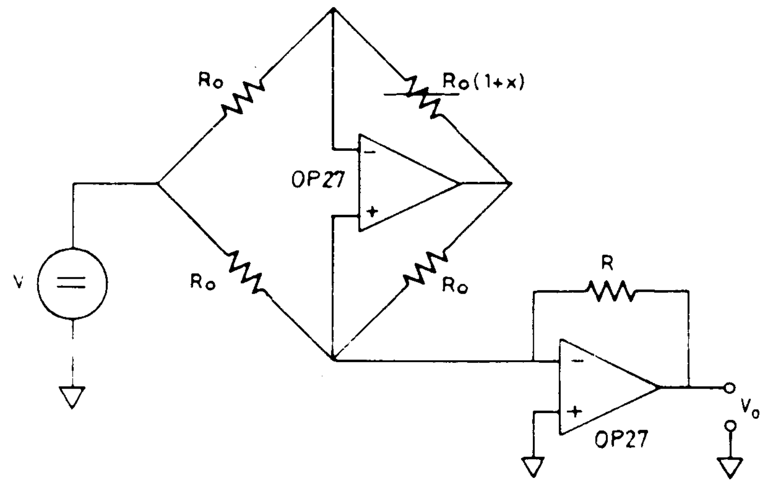

FIGURE 3.23 Analog linearization of a resistance bridge by means of two op amps.

bridge terminals accessible. That is, the bridge must be opened at one of the junctions where the sensor is connected.

The circuit in Figure 3.23 overcomes that limitation at the expense of an additional op amp [5]. The output is

$$V_o = V \frac{R}{R_0} x \tag{3.36}$$

The op amp inserted in the bridge must have low offset voltage and current and their drifts because they are amplified by the second op amp.

3.4.3 Sensor Bridge Calibration

Equation (3.31) shows that the sensitivity of a sensor bridge depends on the supply voltage V, on the sensor's resistance under no load conditions R_0, and on the ratio k between the resistance of the arms. To avoid the need to measure k, which may require opening all the bridge junctions, we can use the arrangement shown in Figure 3.24.

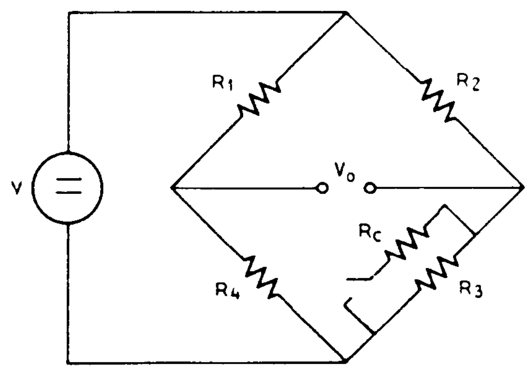

FIGURE 3.24 Calibration of a bridge that includes a resistive sensor.

With the switch opened, for $x = 0$, the bridge is adjusted until $V_o = 0$. After closing the switch with no load applied to the sensor, the output deflection is similar to that obtained from a change x in R_3

$$\frac{R_0 R_c}{R_0 + R_c} = R_0(1 + x) \tag{3.37}$$

$$x = -\frac{R_0}{R_0 + R_c} \tag{3.38}$$

The bridge sensitivity is then

$$S = \frac{V_o}{xR_0} = -\frac{V_o}{R_0}\left(1 + \frac{R_c}{R_0}\right) \tag{3.39}$$

We need to measure only R_0 and the calibration resistor in order to calculate the bridge sensitivity from the measurement of V_o.

If the bridge has more than one active arm, other calibration resistors can be connected one at a time by closing the respective switch, for example, with a relay in an automatic system.

If the calibration resistor or resistors cannot be placed close to the respective sensors, we must avoid placing lead wires for those resistors in series with the sensors. That is, we should use separate wires.

Measurement conditions may be different from calibration conditions, resulting on a nonzero bridge output at null conditions. This problem can be solved by a modified bridge [14], which consists of adding two known resistors R in series with each of R_3 and R_4. Either the junction between R and R_3 or R and R_4 is driven by a current I so that the bridge is rebalanced at null at the measurement conditions. I is derived by a feedback system from the bridge output at null conditions, set either manually or under computer control.

3.4.4 Difference and Average Measurements: Compensation

An additional advantage of a bridge, as compared with a voltage divider, is that it permits the measurement of the difference between quantities or its average. Furthermore it permits an increase in sensitivity by using several sensors and it compensates for interference.

Consider, for example, the circuit in Figure 3.25. The presence of two sensors in adjacent arms permits the measurement of the difference between the sensed parameters because the output voltage is

$$V_o = V\frac{k(x_1 - x_2)}{(k + 1 + x_1)(k + 1 + x_2)} \tag{3.40}$$

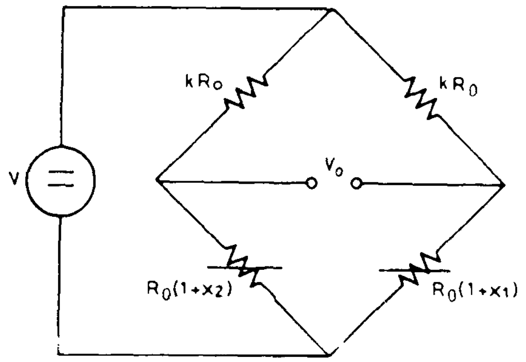

FIGURE 3.25 Measurement of a difference by means of a resistance bridge.

Whenever $x_1, x_2 \ll k + 1$, we can approximate

$$V_o \approx V \frac{k}{(k+1)^2}(x_1 - x_2) \tag{3.41}$$

For temperature sensors this method makes it possible to measure temperature differences. The circuit is useful for the calculation of thermal gradients or heat loss in pipes, or to detect freezing temperatures in agriculture.

A comparison with equation (3.32) shows that the same compromise between sensitivity and linearity holds here, which influences the choice of the value for k.

Example: A differential thermometer able to measure temperature differences from 0°C to 750°C in the range from 50°C to 800°C with an error smaller than 5°C is needed to measure the heat insulation capability of several materials. The sensors available are platinum probes having 100 Ω at 0°C, temperature coefficient 0.4%/°C, and thermal dissipation constant δ of 1 mW/K. Design a dc bridge supplied by a constant voltage, able to perform the desired measurements, and whose output voltage is measured by an ideal voltmeter. What is the sensitivity for the bridge designed?

Using the notation in Figure 3.25 and taking $x_1 = \alpha T_1$, and $x_2 = \alpha T_2$, equation (3.40) leads to

$$V_o = V \frac{k\alpha(T_2 - T_1)}{(k + 1 + \alpha T_1)(k + 1 + \alpha T_2)}$$

The ideal sensitivity is the quotient between output voltage V_o and temperature difference $T_2 - T_1$ if the output were linear,

$$S = \frac{Vk\alpha}{(k+1)^2}$$

The absolute nonlinearity error referred to the input (temperature difference) is

$$e = \frac{V_o}{S} - (T_2 - T_1)$$

$$= -(T_2 - T_1)\frac{(k+1)\alpha(T_1 + T_2) + \alpha^2 T_1 T_2}{(k+1+\alpha T_2)(k+1+\alpha T_1)}$$

Therefore the error increases with the temperature difference. The worst case is thus when $T_1 = 50°C$ and $T_2 = 800°C$. We avoid exceeding the prescribed 5°C limit, and assume that the self-heating error is negligible, then we have

$$e = 750\,\frac{(k+1)4 \times 10^{-3} \times 850 + 16 \times 10^{-6} \times 4 \times 10^{4}}{(k+1+3.2)(k+1+0.2)} < 5$$

which leads to

$$k^2 - 504.6k - 601 = 0$$

The solutions are $k = 506$ and $k = -1.16$. Obviously the negative solution is not valid. The values for the bridge resistors are then $R_1 = R_2 = 50.6\ k\Omega$.

The supply voltage must be low enough to yield a very low power dissipation in the probes. Because the measurement is differential, there will be a self-heating error whenever each probe dissipates a different amount of power. The absolute error, in temperature, is given by

$$e_a = \left(\frac{V}{R_1 + R_4}\right)^2 \frac{R_4}{\delta} - \left(\frac{V}{R_2 + R_3}\right)^2 \frac{R_3}{\delta}$$

$$= \frac{V^2}{R_0\delta}\left[\frac{1+\alpha T_2}{(k+1+\alpha T_2)^2} - \frac{1+\alpha T_1}{(k+1+\alpha T_1)^2}\right]$$

Because k is large, we can approximate

$$e_a \approx \frac{V^2}{R_0\delta}\,\frac{\alpha(T_2 - T_1)}{(k+1)^2}$$

If we want this error to be, for example, only 0.05°C, we can use this equation to deduce that the supply voltage must be less than 20.7 V. Then the sensitivity would be 163 μV/°C. If a lower voltage were chosen for the supply voltage, this error would decrease, and by increasing k a little, it would be possible to keep the total absolute error lower than the intended 5°C limit.

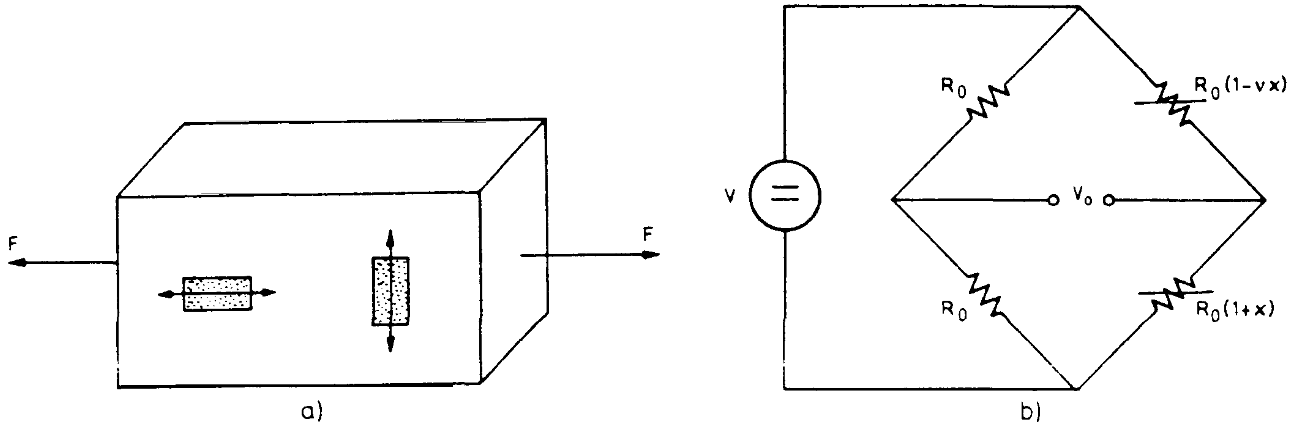

FIGURE 3.26 Use of two active strain gages, one longitudinal and the other one transverse, and its arrangement in a measurement bridge.

The arrangement of several strain gages in the same bridge also offers many advantages. If, for example, two strain gages are bonded to an element as shown in Figure 3.26*a*, and are connected in a bridge as in Figure 3.26*b*, the output voltage is

$$V_o = V\frac{x(1+\nu)}{2[2+x(1-\nu)]} \approx V\frac{x(1+\nu)}{4} \tag{3.42}$$

where we note that the sensitivity has increased by a factor of $1+\nu$ with respect to the case where a single gage is used.

If use is made of two strain gages undergoing strains of the same magnitude but opposite in sign, when connected as shown in Figure 3.27 the output voltage is

$$V_o = V\frac{x}{2} \tag{3.43}$$

In this case not only is the output larger than for one gage, but it is linear.

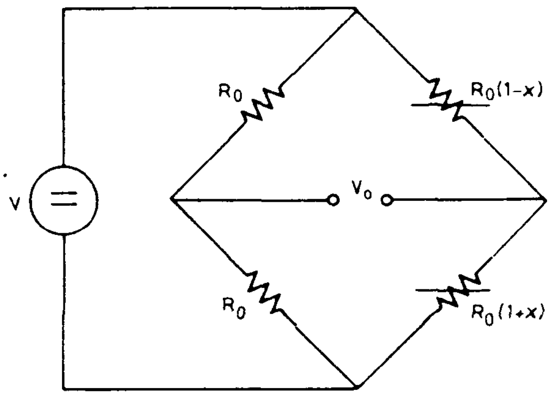

FIGURE 3.27 Arrangement in a measurement bridge of two active strain gages undergoing opposite variations.

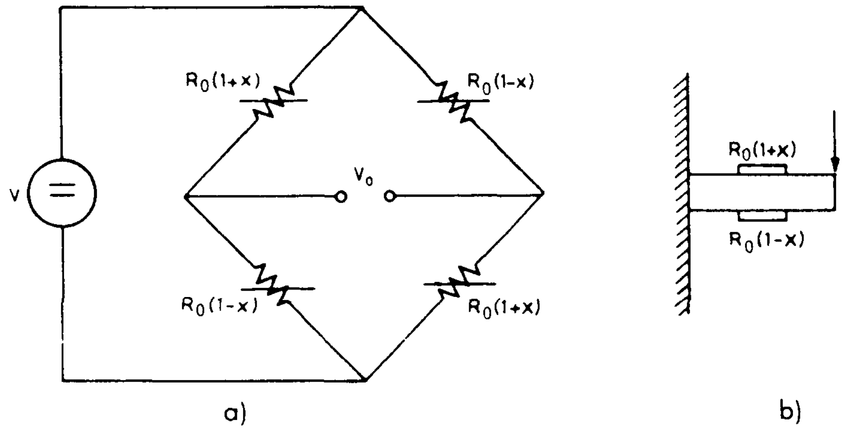

FIGURE 3.28 Bridge linearization by means of dual rosettes.

The circuit shown in Figure 3.28*a* corresponds to the cantilever beam in Figure 3.28*b* where two equal dual rosettes (closely spaced gage combinations) have been bonded on each side. The output voltage is

$$V_o = Vx \tag{3.44}$$

which is also linear and yields a sensitivity four times that of a single strain gage.

These different measurement arrangements are called quarter bridge, half bridge, and full bridge. Table 3.2 compiles the corresponding output voltages for the different arrangements and for constant voltage or constant current supplies. Full bridge arrangements are common in load cells.

TABLE 3.2 Output Voltage for Different Strain Gage Arrangements of Quarter Bridge, Half Bridge, and Full Bridge (see Figure 3.19), Supplied by a Constant Voltage or Current

R_1	R_2	R_3	R_4	Constant V	Constant I
R_0	R_0	$R_0(1+x)$	R_0	$V\dfrac{x}{2(2+x)}$	$IR_0\dfrac{x}{4+x}$
$R_0(1+x)$	R_0	$R_0(1+x)$	R_0	$V\dfrac{x}{2+x}$	$IR_0\dfrac{x}{2}$
R_0	R_0	$R_0(1+x)$	$R_0(1-x)$	$V\dfrac{2x}{4-x^2}$	$IR_0\dfrac{x}{2}$
R_0	$R_0(1-x)$	$R_0(1+x)$	R_0	$V\dfrac{x}{2}$	$IR_0\dfrac{x}{2}$
$R_0(1-x)$	R_0	$R_0(1+x)$	R_0	$V\dfrac{-x^2}{4-x^2}$	$IR_0\dfrac{-x^2}{4}$
$R_0(1+x)$	$R_0(1-x)$	$R_0(1+x)$	$R_0(1-x)$	Vx	IR_0x

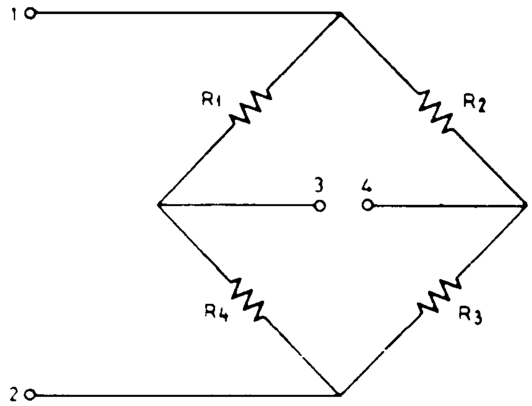

FIGURE E3.3 Resistor and terminal labeling for a load cell.

Example: Figure E3.3 shows a strain-gage-based load cell which seems to have been damaged. It is not possible to obtain a zero output under no load conditions. To find a possible explanation, the voltage supply leads (1 and 2) and amplifier leads (3 and 4) are disconnected. Then several measurements between leads are carried out that yield the following results: between 1 and 2, 127 Ω; between 1 and 3, 92 Ω; between 1 and 4, 92 Ω; between 2 and 3, 92 Ω; between 2 and 4, 106 Ω; between 3 and 4, 127 Ω. All the measurements are performed with open connections for the nonmeasured lead pair. Determine which is the bad gage and give a possible explanation for its damage.

Using the terminology in Figure E3.3, the measurements carried out are of the following resistance combinations

$$R_{12} = \frac{(R_1 + R_4)(R_2 + R_3)}{R_1 + R_2 + R_3 + R_4} = 127\ \Omega$$

$$R_{13} = \frac{R_1(R_4 + R_2 + R_3)}{R_1 + R_2 + R_3 + R_4} = 92\ \Omega$$

$$R_{14} = \frac{R_2(R_4 + R_1 + R_3)}{R_1 + R_2 + R_3 + R_4} = 92\ \Omega$$

$$R_{23} = \frac{R_4(R_1 + R_2 + R_3)}{R_1 + R_2 + R_3 + R_4} = 92\ \Omega$$

$$R_{24} = \frac{R_3(R_1 + R_2 + R_4)}{R_1 + R_2 + R_3 + R_4} = 106\ \Omega$$

$$R_{34} = \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} = 127\ \Omega$$

From the first and sixth results we deduce $R_2 = R_4$. From the second and third, $R_1 = R_2$. The fourth confirms that $R_1 = R_4$. The fifth indicates that R_3 is different. The problem reduces thus to find two resistances, R and R_3, for which two equations are enough

$$\frac{R(2R + R_3)}{3R + R_3} = 92$$

$$\frac{3RR_3}{3R + R_4} = 106$$

By solving these equations, we obtain $R = 120\ \Omega$ and $R_3 = 150\ \Omega$. This large value for R_3 and the inability of nulling the zero may be due to an overload of R_3 and a nonreversible (permanent) deformation.

Strain gages are temperature sensitive, and a bridge minimizes this problem. If a single strain gage is used that undergoes a relative change y because of a temperature change in addition to a relative change x due to the stress to be measured, all that is necessary in order to compensate for the temperature-induced changes is to use an additional similar dummy gage (not subjected to stress) and arrange them as shown in Figure 3.29. When multiple gages are used, thermal compensation exists, so we do not require the dummy gage.

Because of resistor tolerance, when we construct a Wheatstone bridge, we do not usually obtain the balance condition given by (3.30). To obtain the balance condition, we can use the circuit shown in Figure 3.30.

The last application for Wheatstone bridges is the measurement of average values, as shown in Figure 3.31. We assume that the three sensors (a large number could be used) are all equal but measure different values of the same quantity, such as temperature. The output voltage is then

$$V_o = V\frac{k(x_1 + x_2 + x_3)/3}{[k + 1 + (x_1 + x_2 + x_3)/3](k + 1)} \tag{3.45}$$

That is, the output voltage is proportional to the average value if $k + 1$ is large enough.

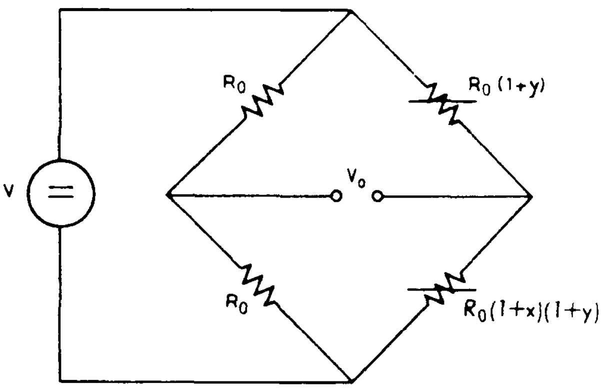

FIGURE 3.29 Temperature compensation in a strain gage bridge by using a dummy gage.

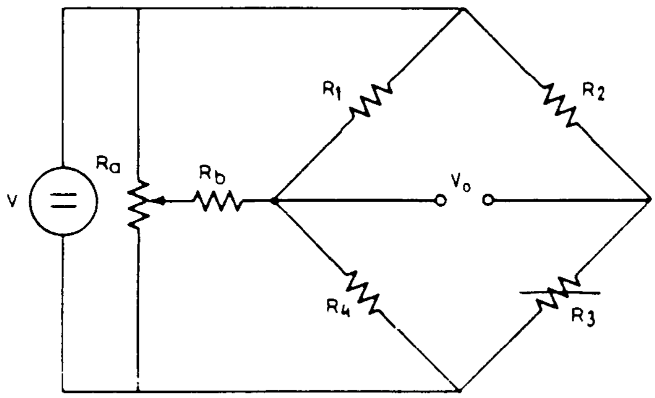

FIGURE 3.30 Circuit for the initial balancing of a resistance sensor bridge.

3.4.5 Power Supply of Wheatstone Bridges

To obtain an output signal from a Wheatstone bridge as a result of the change in one or more sensors placed in its arms, we must supply the bridge with electric excitation. Whether voltage or current, dc or ac, this supply must be stable with time and temperature. This is due to a simple fact: When a resistive bridge is supplied by a dc voltage, for example, the output voltage is (Equation 3.31)

$$V_o = V \frac{kx}{(k + 1)(k + 1 + x)}$$

Now if while x remains constant, V undergoes a small change dV. Then we have

$$\frac{dV_o}{V_o} = \frac{dV}{V} \tag{3.46}$$

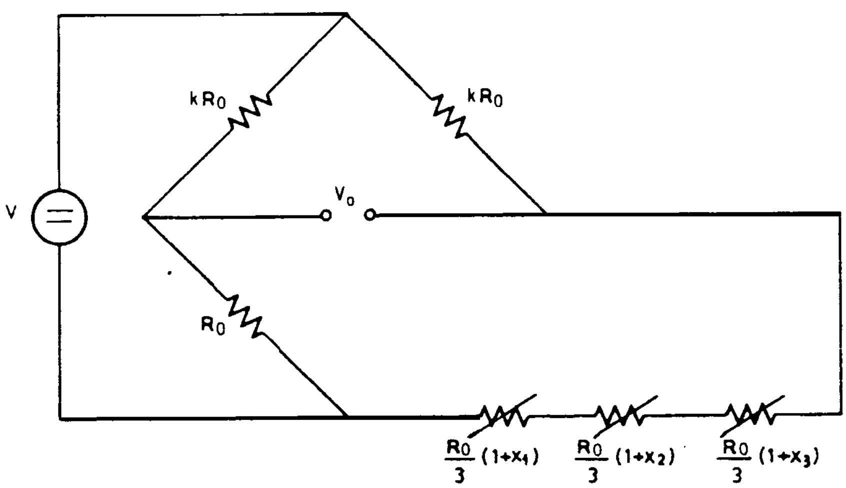

FIGURE 3.31 Measurement of average values by means of a resistance bridge. All sensors must have the same nominal resistance and sensitivity.

TABLE 3.3 Stability of Several Components that Provide a Constant Output Voltage

	Designation			
Drifts	AD 580	2B35	REF 01	LM 399A
Time	$25 \times 10^{-6}/1000$ h	10^{-4}/mo	—	$2 \times 10^{-5}/1000$ h
Thermal	$30 \times 10^{-6}/°C$	$5 \times 10^{-4}/°C$	$5 \times 10^{-7}/°C$	$10^{-6}/°C$
Supply	$50 \times 10^{-6}/V$	$8 \times 10^{-4}/V$	$12 \times 10^{-5}/V$	4.2 mV/mA

which means that the output undergoes the same percent change. This may preclude, for example, the use of an ordinary power voltage supply with a 0.1%/°C drift or that of some monolithic voltage regulators with thermal drifts in excess of 1%/°C.

In applications where high precision is required, we must use high-quality ac/dc or dc/dc converters or reference voltage generators like the ones used in multiplying DACs (digital-to-analog converters). Table 3.3 gives the stability characteristics for several of these components.

Another factor to be considered is the maximal output current of the supply unit. Reference voltage generators used in ADCs and DACs, for example, source a maximal current lower than 20 mA and a voltage of +10 V. Therefore they can be directly applied only to bridges having a 500-Ω or higher resistance. If larger voltage or current is needed then the supply output must be amplified without degrading its stability. Figure 3.32 shows a circuit proposed for this application [6].

The need for a high stability for the supply voltage does not apply if the bridge output voltage is further processed by dividing it by the reference voltage. If the same voltage is used for supply and as a reference, then their possible drifts cancel. These kinds of measurements are called *ratiometric.*

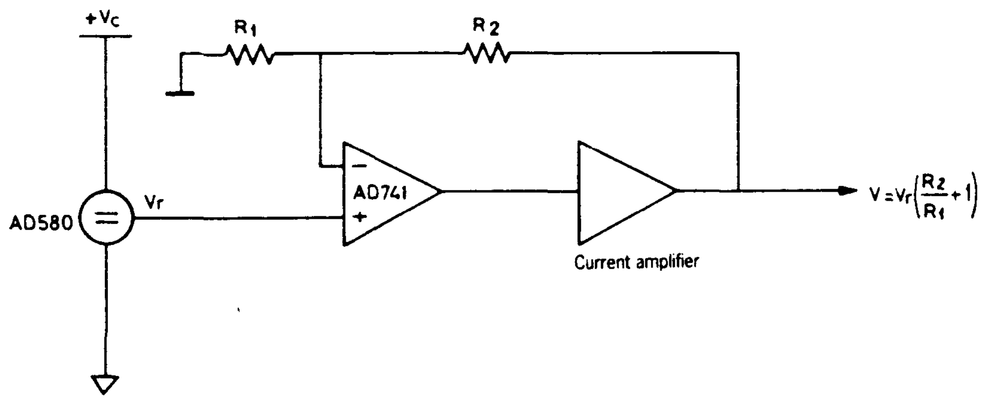

FIGURE 3.32 Circuit to amplify the output current provided by a high-stability reference voltage generator.

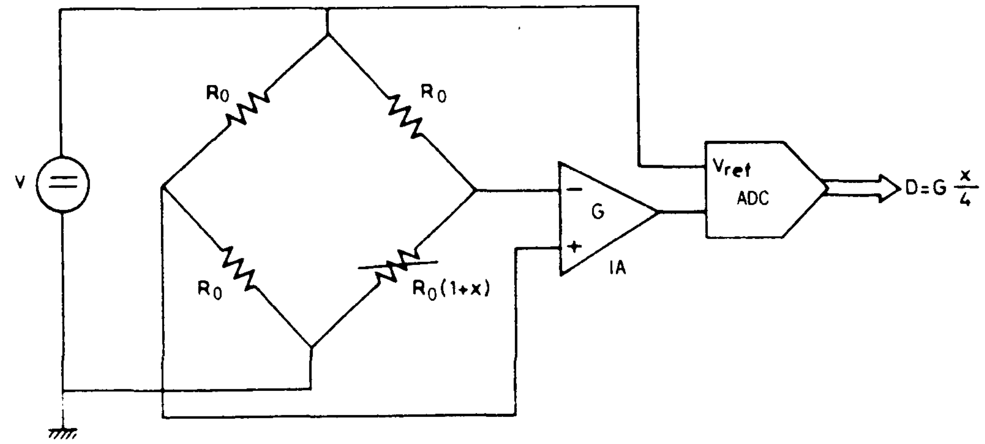

FIGURE 3.33 Ratiometric measurements based on an analog to digital converter eliminates the need for a high stability of the bridge supply.

This measurement principle can be applied, for example, when the processing circuitry includes an ADC because an ADC operates as a divider with digital output. Figure 3.33 shows that the input voltage to the converter is compared with the reference voltage; that is, it is divided by it. If the voltage supply for the bridge were ac, the same method could be used, but the ac voltage would have to be rectified in order to yield the reference voltage. The voltage supply for the amplifier does not require high stability because the amplifier is able to reject its variations, as given by the specified PSRR (Power Supply Rejection Ratio).

Another supply-related problem arises when a low-resistance bridge is so far away that lead wire resistances can no longer be neglected. The problem does not arise if the bridge is supplied by a current instead of a voltage. But usually it is more cumbersome to have a low drift current source than a voltage source. For a voltage supply the solution relies on applying the four wire method shown in Figure 3.34.

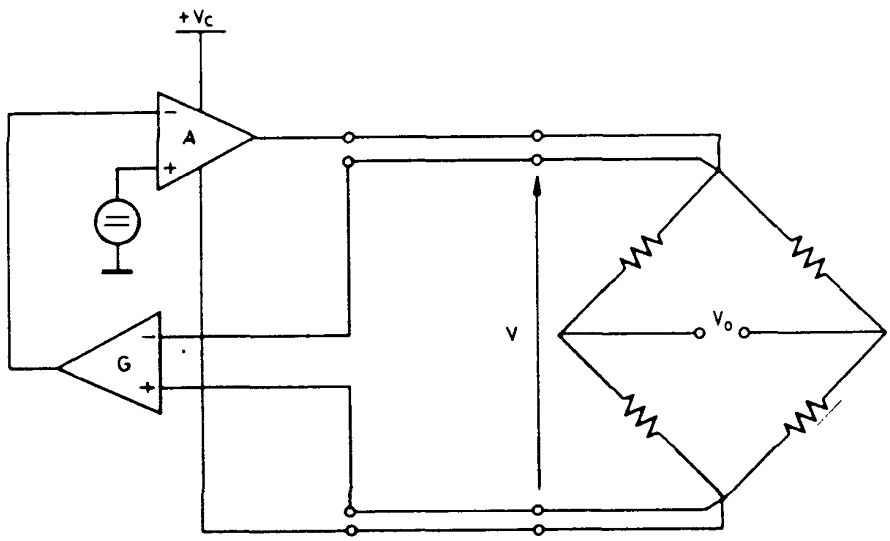

FIGURE 3.34 Four-wire measurement method to compensate for the drop in voltage along supply wires to a remote bridge.

It consists of applying the voltage with two wires and detecting the drop in voltage across the bridge using a different pair of wires; after amplification (G) the detected voltage is used to adjust the voltage output from the source by means of a high-gain amplifier (A). Note that this method does not avoid the drop in voltage along the supply lead wires but only yields the desired supply voltage across the bridge. Therefore the power supply will have to provide the bridge voltage plus the voltage drop along the wires. For the particular case of reference voltage generators that accept an error signal in the form of a current, it is possible to implement a circuit of this kind by using only three wires as shown in Figure 3.35 [7].

A last consideration relative to bridge supplies concerns the choice of a dc or ac signal for excitation. If a dc signal is used, then the thermoelectromotive effects (Section 6.1.1) appearing in junctions of dissimilar metals and amplifier drifts cause errors that restrict the physical layout of the circuit. An ac supply avoids thermoelectromotive effects but stray capacitances may cause bridge imbalance. Between a strain gage and the structure it is bonded on is a capacitance of approximately 100 pF. The impedance of stray capacitances has a more pronounced effect at high frequencies. But the supply frequency cannot be arbitrarily low if we measure dynamic variables as we will find in Chapter 5. Furthermore, if the measurement range includes both positive and negative values, we will require a phase-sensitive detector in order to know the sign of the bridge's output signal. As a result an ac supply is not usually used, except in those applications where the available sensor favors that kind of supply or when we desire the low noise of ac amplifiers.

3.4.6 Detection Methods for Wheatstone Bridges

The type of detecting device for the output signal of a sensor bridge depends on the intended application. Except for those cases of voltage, current, or

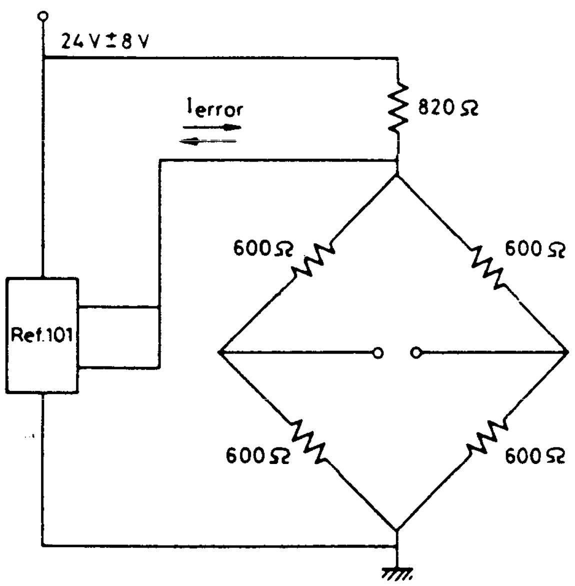

FIGURE 3.35 Bridge supply using three wires based on remote current detection.

frequency telemetry for which specific converters are available, most situations call for either an immediate analog presentation or an analog-to-digital conversion. In any case the detector must have an adequate input impedance—usually high because it is intended to measure the bridge output voltage—and an input arrangement compatible with ground connections in the bridge supply. In particular, the meter input must be differential if the bridge supply is grounded.

The easiest choice for an immediate analog presentation is a galvanometer. It is inherently differential, which is an advantage when compared with other detectors. But its input impedance is medium to low, and it lacks the necessary robustness for industrial applications if it must also be sensitive. Furthermore it is too slow for dynamic measurements, and in general it works only for dc supplies. Although galvanometers were the better choice before the advent of integrated circuits, these shortcomings reduce its application in present measurement systems.

An oscilloscope is an alternative to galvanometers when a dynamic signal is being measured. When no probe is used, it presents a 1-MΩ input resistance, which is high enough for most cases. But unless the bridge power supply is floating and the external interference is small, it must have a differential input. If a high sensitivity is also required, its cost can be very high. An alternative to oscilloscopes are paper and chart recorders, but both require the input signal to be amplified. Therefore we consider these to be presentation methods for amplified signals.

Whether for an immediate digital presentation or for digital transmission or calculation, the analog bridge output signal must be converted into digital form. If a bench or panel voltmeter is used, that function is performed by the instrument, which has an input resistance of 10 MΩ or higher. But because of its cost and lack of flexibility, the use of such a subsystem is not always the best solution. Nevertheless, there is an increasing availability of custom-tailored digital panel meters that also offer a simultaneous digital output signal for remote connection.

Amplification techniques suitable to convert low-amplitude bridge signals into 1 or 10 V signals, as required by usual ADCs at their inputs, are dealt with in the next section.

3.5 INSTRUMENTATION AMPLIFIERS

3.5.1 Differential Amplifiers

Most resistance sensor bridges are supplied by a grounded voltage or current source. Therefore the amplifier at the bridge's output should not have any of its input terminals grounded. In addition we will show later that it is best for input terminals to have high and similar impedances to ground. An amplifier having these characteristics is called a *differential amplifier*.