Due distribuzioni notevoli in gestione della produzione

La distribuzione di Poisson

La distribuzione esponenziale

La distribuzione di Poisson

- The Poisson distribution is used to determine the probability that a specific number of occurrences takes place within a given unit of time or space. A random variable *X* is Poisson distributed if:

1. The experiment consists of counting the number of times, *x*, an event occurs in a given interval (the interval can relate to time, area or volume).
2. The probability of the event occurring is the same for each interval.
3. The number of occurrences in one interval is independent of the number of occurrences in other intervals.

La distribuzione di Poisson

- Notation:

$$P(X = x) = \frac{m^x e^{-m}}{x!}$$

Note that *x* can be any non-negative integer.

$$\text{Mean}: m \text{ (rate per interval)}$$
$$\text{Variance}: m$$

La distribuzione di Poisson

- The mean catch of dolphins per month in the Eastern Tropical Pacific is 3.
  - What is the probability that the catch will be 4 in a given month?

$$P(X = x) = \frac{m^x e^{-m}}{x!}$$

$$P[X = 4] = \frac{m^x e^{-m}}{x!} = \frac{3^4 e^{-3}}{4!} = 0.168$$

La distribuzione di Poisson

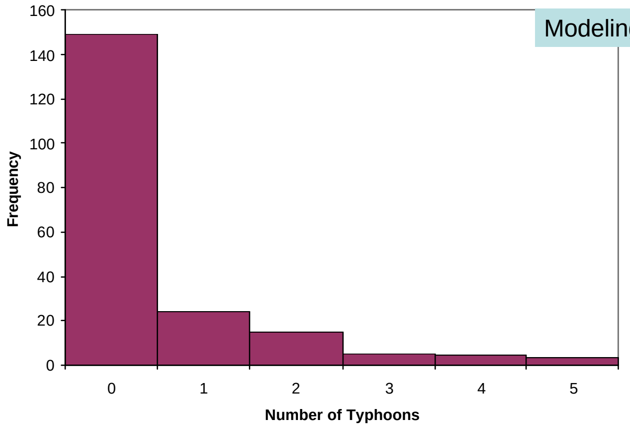
- The mean catch of dolphins per month in the Eastern Tropical Pacific is 3.
  - What is the probability that the catch will be 2 or less in a given month?

$$P(X = x) = \frac{m^x e^{-m}}{x!}$$

P[X≤2]=P[X=0]+P[X=1]+P[X=2]=

$$P[X \leq 2] = e^{-3}[\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}] = 0.423$$

La distribuzione di Poisson



Modeling Typhoons

Assuming that the typhoons are Poisson distributed, find the mean number of typhoons per year and the probability that there will be three or more in a single year.

La distribuzione di Poisson

Modeling Typhoons

- Mean number of typhoons:

$$m = \frac{\sum_i n_i \, x_i}{N} = 0.5$$

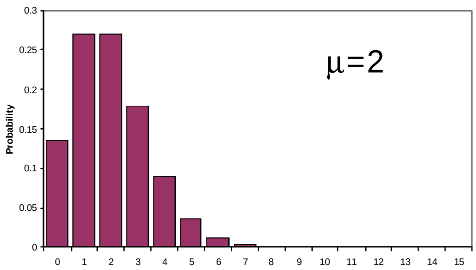
- Probability of *x* typhoons a year:

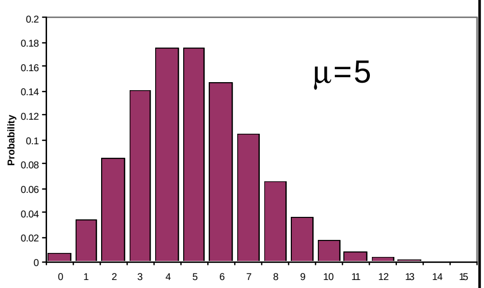
0	1	2	3	4	5	6
149	24	15	5	4	3	
0.607	0.303	0.076	0.013	0.002	0.000	0.000

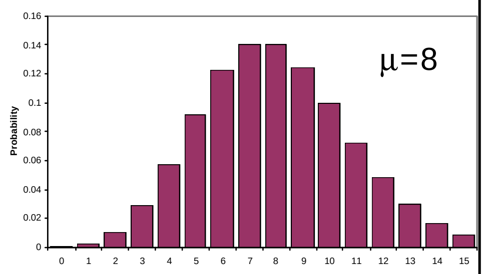
- The probability of the number of typhoons being at least 3 is 0.015.
- Later we will examine methods to determine whether it is reasonable to assume that these data are Poisson distributed.

Distribuzione esponenziale e di Poisson- 7

La distribuzione di Poisson







Distribuzione esponenziale e di Poisson- 8

La distribuzione esponenziale G

- La Exponential distribution trova applicazione nella teoria delle code e negli studi di affidabilità.
  - Time between customer arrivals at a terminal

  - Time to failure of electrical or mechanical components

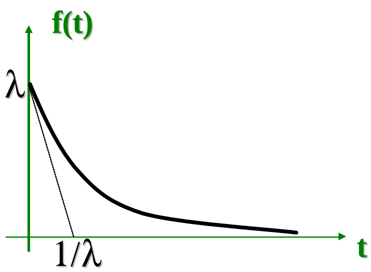
La distribuzione esponenziale G

PDF $f(t) = l \cdot e^{-lt}$

CDF $F(t) = 1 - e^{-lt}$

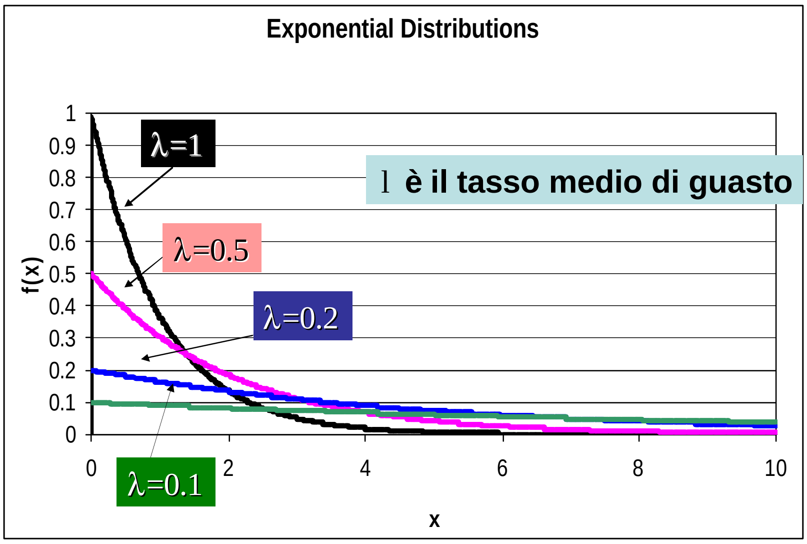
E(X) $m_x = \frac{1}{l}$

Var(X) $s_x^2 = \frac{1}{l^2}$

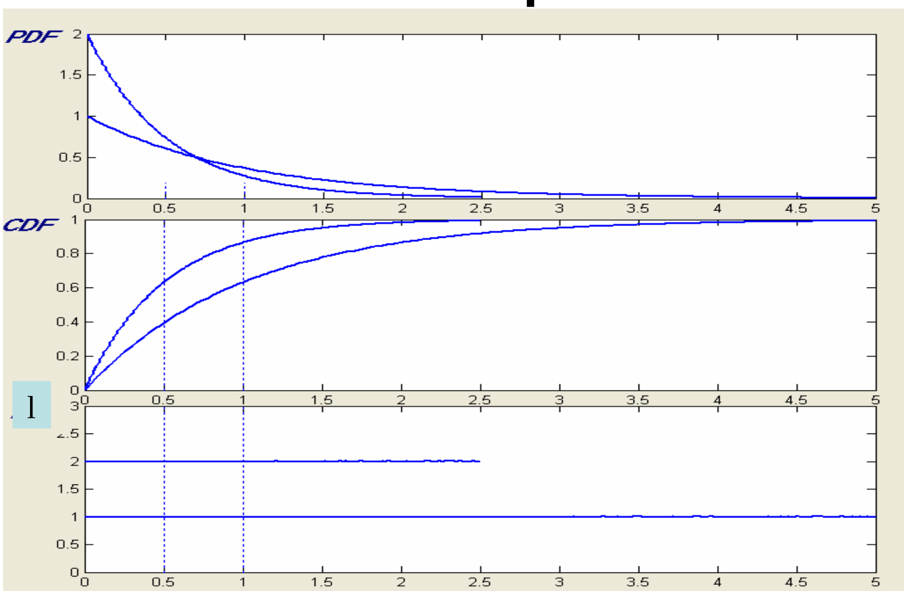


m **è il tempo medio all’evento (guasto)**
l **è il tasso medio di guasto**

La distribuzione esponenziale G



La distribuzione esponenziale G



La distribuzione esponenziale G

- Assenza di memoria

Suppose that *X* is a random variable with the following properties

1. $P[X \geq 0] = 1$
2. $P[x \leq X \leq x + dx \,|\, X \geq x] = l\,dx$

$\Rightarrow X \sim \Gamma\left(\frac{1}{l}, \frac{1}{l^2}\right)$

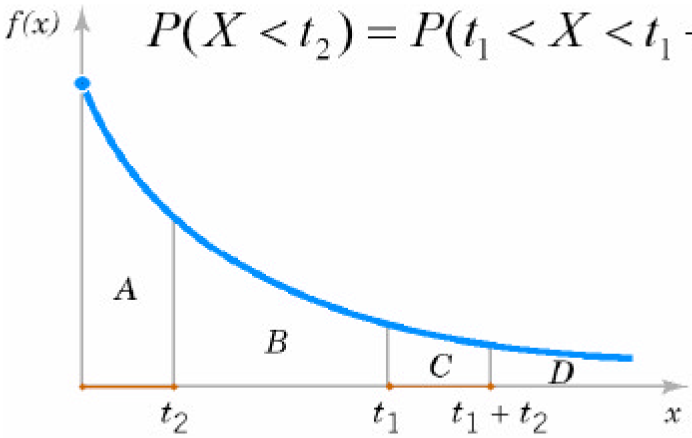
$$P(X < t_2) = P(t_1 < X < t_1 + t_2 \,|\, X > t_1)$$

The exponential distribution is used as a model for the lifetime of an object that doesn't age.

La distribuzione esponenziale G

- Assenza di memoria

$$P(X < t_2) = P(t_1 < X < t_1 + t_2 \,|\, X > t_1)$$



La distribuzione esponenziale G

- *Relationship with Poisson distribution:*

If the lifetime follows exponential distribution, then the number of failures within an interval (0, t) follows the Poisson distribution.

$$\Pr(N(0,t) = x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t}, \quad x = 0, 1, 2, \ldots.$$